Honeycomb composites with auxetic out-of-plane characteristics

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ABSTRACT

Systems with a negative Poisson's ratio (auxetic) exhibit the unusual yet very useful property of getting wider rather than thinner when uniaxially stretched. A novel mechanism to generate auxetic behaviour at tailor-made values which may be implemented in composites manufacture using readily available materials is proposed. FEA simulations are used to provide a proof of principle for this concept and an analytical model is proposed so as to elucidate the requirements for auxetic behaviour. It is shown that the simulations and analytical model give comparable results and confirm that this system may reach giant negative values of the Poisson's ratio.

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1. Introduction

Composite materials (i.e. materials made from more than one component, so designed to obtain the best properties of the different constituents) and cellular solids (porous materials with a high strength to density ratio characteristics) represent two of the more important classes of solids which can be used in various practical applications ranging from aerospace applications to household goods. A number of studies have been published in recent years on such systems, especially on two dimensional honeycombs [1–24] and composites such as sandwich panels which incorporate cellular cores [25–32] and two-phase cellular structures [32–39]. A number of two dimensional honeycombs have also been studied for their ability to exhibit a negative Poisson's ratio [14–24]. In such cases the system expands in the lateral direction when uniaxially stretched, a phenomenon which is more commonly known as auxetic behaviour [40–49]. For such systems, it has been shown that negative Poisson's ratios depend on the geometry of the honeycomb and the mechanism by which it deforms upon application of stress. In fact, re-entrant honeycombs deforming through hinging and non re-entrant honeycombs deforming through stretching of the ribs exhibit a negative Poisson's ratio while, re-entrant honeycombs deforming through stretching of the ribs and non re-entrant honeycombs deforming through hinging exhibit a positive Poisson's ratio [11], see Fig. 1.

Composite systems have also been studied for their potential to exhibit auxetic behaviour. Such systems include laminates of fibre-reinforced composites which are made from materials having a positive Poisson's ratio that exhibit a negative through-thickness Poisson's ratio when the layers are oriented in a specific direction with respect to one another [50–52]. Other studies on the Poisson's ratios of composites have focused on the effect that changing specific parameters, such as; fibre type, density and orientation, resin type and ply orientation had on the through thickness Poisson's ratios [53–63]. Also, more recent studies have explored the idea of laminated composites made from alternating conventional and auxetic laminas, resulting in a through-thickness Young's modulus higher than that predicted by the rules of mixture [64,65].

Auxetic honeycombs and composites (and other auxetic systems in general), are thought to perform better in a number of applications, due to their superior properties. For example, auxetic systems have been shown to provide better indentation resistance [66–68], have the ability to form dome shaped structures when bent [69] and have better acoustic and vibration properties over their conventional counterparts [70–73].

This work studies a two-phase cellular system similar to that studied by Evans et al. [32] *vis-à-vis* its potential to exhibit auxeticity. The auxetic composite system is made up of a conventional honeycomb framework where its pores are filled with a much softer matrix that is forced out when the honeycomb is pulled, thus producing the required auxetic effect. We use Finite Elements simulations to verify the potential of such systems to exhibit the proposed out-of-plane auxetic behaviour and also to study the effect that changes in the framework geometry and the Poisson's ratio of the matrix have on both the in-plane and out-of-plane Poisson's ratio of the composite in attempt to optimise the parameters of the composite for maximum auxetic behaviour.

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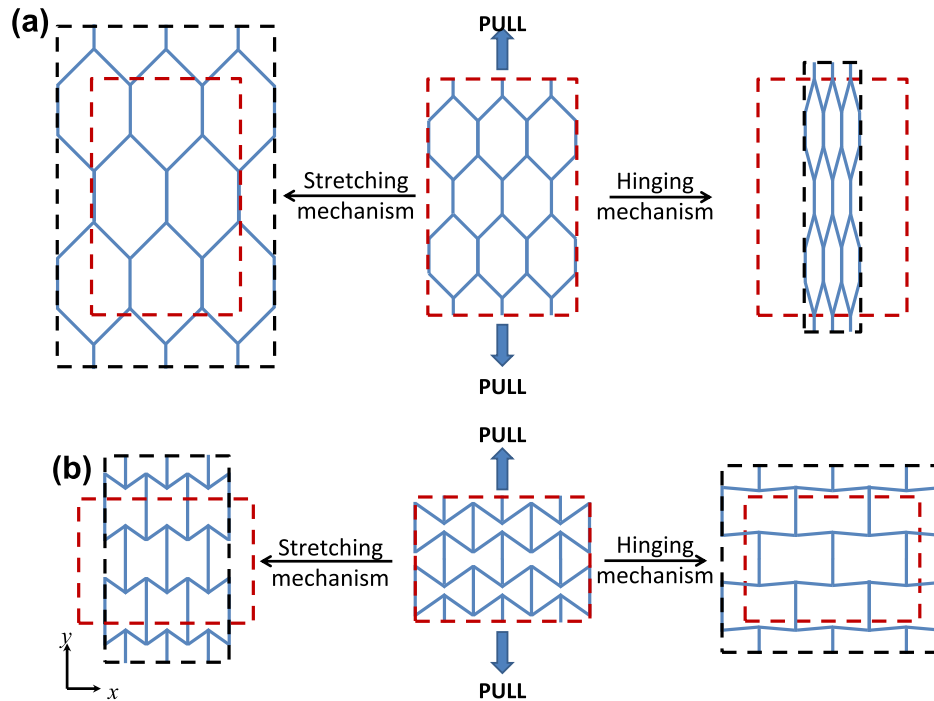 E-mail address: joseph.grima@um.edu.mt (J.N. Grima).

Fig. 1. (a) Non-re-entrant honeycomb deforming through hinging (exhibiting a positive Poisson's ratio), and stretching (exhibiting auxetic behaviour) and (b) re-entrant honeycombs deforming via hinging (exhibiting a negative Poisson's ratio) and stretching, exhibiting a positive Poisson' ratio.

2. The proposed model

The proposed system is based on the idea that if the pores in a honeycomb-like system are filled with a material that bulges out of the pores when the honeycomb is stretched, then the system would exhibit a negative Poisson's ratio in the plane orthogonal to that of the honeycomb. It is envisaged that for this concept to operate, if the matrix is made of a conventional much softer rubber-like material, one would need a honeycomb-like system where the pores shrink considerably as the system is pulled, a property which is easily manifested by the conventional non re-entrant hexagonal honeycombs or wine-rack like systems, as illustrated in

Fig. 2. It is also envisaged that the extent of the magnitude of Poisson's ratio of this composite will depend, amongst other things, on the extent of the change in volume of the pores upon uniaxial stretching as well as the mechanical properties of the matrix. In particular, for maximum out of plane auxeticity, one would require a matrix which bulges out to the maximum as the pores change shape and size. If the matrix had to be made from isotropic materials, in the event that the pores would be getting smaller upon stretching (as one would expect to happen in classical non re-entrant honeycombs), then the ideal matrix material would obviously be one having Poisson's ratios $c.$ +0.5, a property which is easily attainable by rubbery like materials. Conversely, in the event that
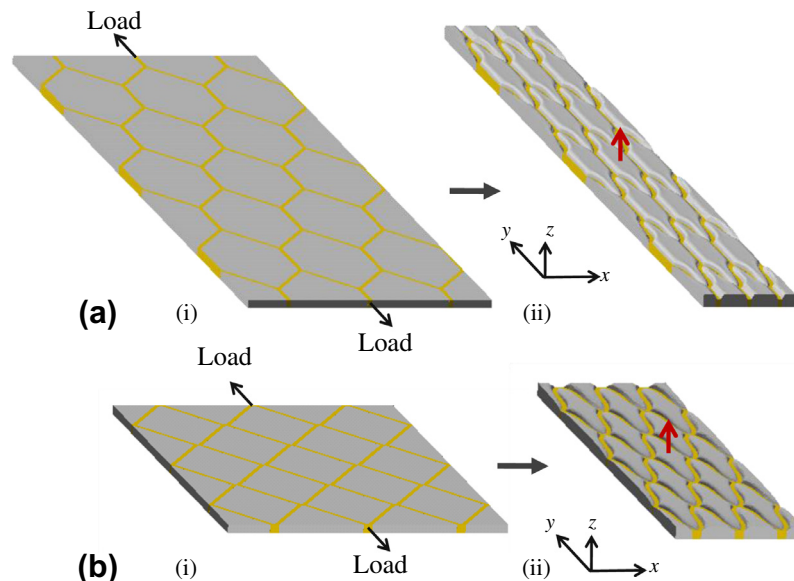

Fig. 2. Depiction of the proposed honeycomb-like system filled with a soft matrix, (a) a non-re-entrant honeycomb system and (b) a wine-rack system (i) un-deformed and (ii) deformed systems with a magnified displacement scaling illustrating deformations of the matrix which bulges out to lead to the observed auxetic effect.

the pore would have been getting larger upon stretching, it is envisaged then one would require materials having negative Poisson's ratios, something which may be more difficult to achieve. It is also envisaged that other properties of the system would play a role, such as the relative stiffness of the material of the honeycomb and that of the matrix, where for optimal performance it will be assumed that the matrix is made from a much softer material than that of the honeycomb.

3. Simulations

In an attempt to verify that auxetic behaviour can indeed be obtained from the systems proposed here, as well as study the extent of auxeticity, finite Element modelling was carried out using the commercially available software ANSYS, V13. Referring to Fig. 3a the composite modelled here was constructed by first building a honeycomb structure made from an isotropic material having a Young's modulus E_S and a Poisson's ratio v_S. This honeycomb was aligned parallel to the xy-plane with the ribs aligned to the vertical (y) axis having a dimension of h cm and the slanting ribs, aligned at an angle θ to the horizontal (x) axis, having a length of l cm. Each rib had an in-plane thickness of t cm and a depth of z cm. The honeycomb structure was then embedded in a much softer matrix having Young's modulus E_M and a Poisson's ratio v_M. The composite was meshed using the 3D solid, 4 node tetrahedral SOLID187 elements, where for each structure generated a convergence test was performed. More specifically, the matrix and the honeycomb were meshed using a different element size specific to each structure, with the maximum element size of $t/3$ and a minimum size determined by the smart sizing option (set to 4) provided by ANSYS. Boundary conditions were then applied to the system as shown in Fig. 3b where, for loading in the y-direction: the nodes on the xy-plane with a minimum value of z were constrained not to move in the z-direction, the nodes in the yz-plane coincident with line a were constrained not to move in the x-direction; the nodes in the xz-plane coincident with line b were constrained not to move in the y-direction while those coincident with line d were coupled to move together in the x-direction. In an attempt to simulate stretching of the systems in the y-direction, a 0.05 engineering strain was applied on the nodes in the yz-plane coincident with line c. Equivalent boundary and loading conditions were applied for stretching in the x-direction. To measure the dimension changes in the directions orthogonal to the direction of stretching, the average displacements in the orthogonal directions were measured from which the strains where calculated.
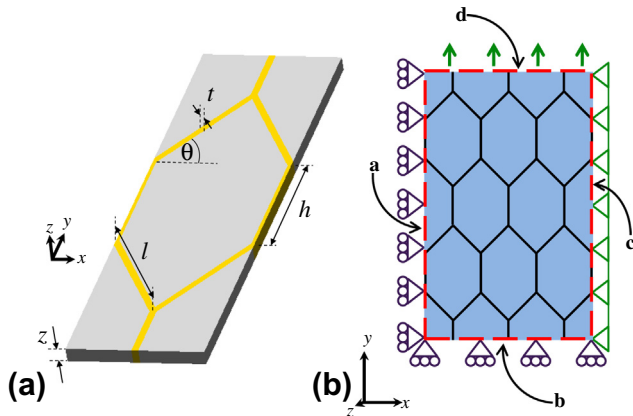
Here, it should be noted that in the case of the out-of-plane direction, this averaging was essential as no coupling was assumed, i.e. the different nodes could move independently of each other.

Simulations were then performed with the aim of analysing the effect that changes in specific geometrical properties of the honeycomb framework have on the in-plane and out-of-plane properties of the system. Three sets of simulations were performed, varying one property at a time, as follows:

(a) $l = 10$ cm, $h \in \{1, 7, 14, 20\}$ cm, $t = 0.5$ cm and $z = 1$ cm;
(b) $l = 10$ cm, $h = 10$ cm, $t \in \{0.3, 0.5, 0.7, 1.0\}$ cm and $z = 1$ cm;
(c) $l = 10$ cm, $h = 10$ cm, $t = 0.5$ cm and $z \in \{1, 4, 7, 10\}$ cm.

where in all of these cases, the simulations were performed for re-entrant structures with $\theta \in \{-25°, -20°, \cdots, -5°\}$ and non re-entrant structures with $\theta \in \{10°, 20°, \ldots, 60°\}$, whenever the system was physically realisable. The mechanical properties of the material making up the honeycomb framework were assumed to be isotropic having a Young's modulus (E_S) of 2×10^{12} Pa and a Poisson's ratio (v_S) of 0.3 whilst the mechanical properties of the matrix were assumed to be isotropic with a Young's modulus (E_M) of 7×10^5 Pa and a Poisson's ratio (v_M) of 0.49.

Additional simulations were also performed with the aim of studying the effect that the Poisson's ratio of the matrix has on the in-plane and out-of-plane Poisson's ratio of the composite structure. For this second part of the study, the geometry of the framework had fixed values of $l = 10$ cm, $h = 10$ cm, $t = 0.5$ cm, $z = 1$ cm and $\theta \in \{-25°, -20°, \ldots, -5°, 10°, 20°, \ldots, 60°\}$. The Young's modulus and Poisson's ratio of the honeycomb structure were set to $E_S = 2 \times 10^{12}$ Pa and $v_S = 0.3$ respectively, while the Young's modulus of the matrix was set to $E_M = 7 \times 10^5$ and the Poisson's ratio was varied as $v_M \in \{-0.95, -0.65, -0.35, 0, 0.25, 0.35, 0.45\}$.

4. Results and discussion

Images showing the unstretched and stretched conformations of typical honeycombs are shown in Fig. 2. Also shown in Fig. 4 are plots of the simulated on-axis in-plane and out-of-plane Poisson's ratios against θ as well as the out-of plane Poisson's ratio against the in-plane Poisson's ratio for a typical composite system. Further results for other cases considered are presented as Supplementary information. These images and plots clearly confirm that, provided that for the systems considered here where the modulus of the matrix is much smaller than the modulus of the system, composite systems which have a high positive in-plane Poisson's ratio can exhibit a negative out-of-plane Poisson's ratio, which effect is caused by the bulging out of the soft matrix from the honeycomb pores as these close when the honeycomb is stretched. Note that the systems modelled here can be easily developed from readily available materials and components, such as those used in typical metal honeycombs and for soft rubbery matrices.

In fact, if one had to look at the behaviour of all the systems considered having matrix Poisson's ratios of c. +0.5, then, irrespective of the direction of loading or geometric parameters used, the out-of-plane Poisson's ratio appears to be either c. zero, something which happens when the in-plane Poisson's ratio of the structure is less than a threshold value (which in this case seems to be c. +1), or, related to the in-plane Poisson's ratio through a linear relationship (which in this case seems to be one with a gradient of c. −1), as indicated in Fig. 5. Similar trends, discussed in detail below, are obtained when the matrix Poisson's ratio is varied, although in this case the gradient of the plot between the in-plane and out-of-plane Poisson's ratios depends on the Poisson's ratio of the matrix (see Fig. 6).
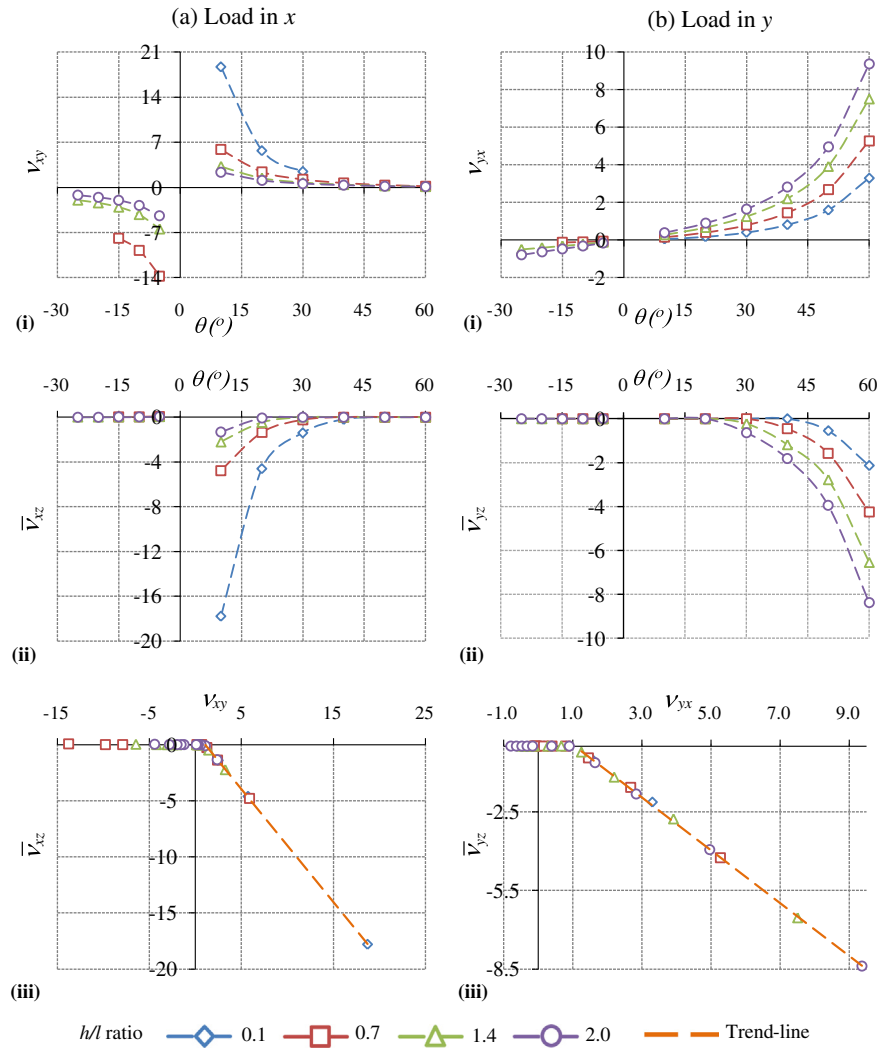

Fig. 3. (a) The modelled composite showing the defined parameters for building the structure and (b) the constraints used for simulating the composite when loading along y.

Fig. 4. Plots showing the variation of (i) on-axis in-plane and (ii) out-of-plane Poisson's ratios against θ as well as (iii) the out-of-plane Poisson's ratio against the in-plane Poisson's ratio when loading typical composite systems having different h/l aspect ratios of the honeycomb framework (with $l = 10$ cm, $h \in \{1, 7, 14, 20\}$ cm, $t = 0.5$ cm and $z = 1$ cm) along the (a) x- and (b) y directions. The data for (aiii) and (biii) fit a trend line having $y = -1.0051x + 1.0487$ with an $R^2 = 0.9999$ and $y = -1.0068x + 1.0624$ with an $R^2 = 0.9995$ respectively.
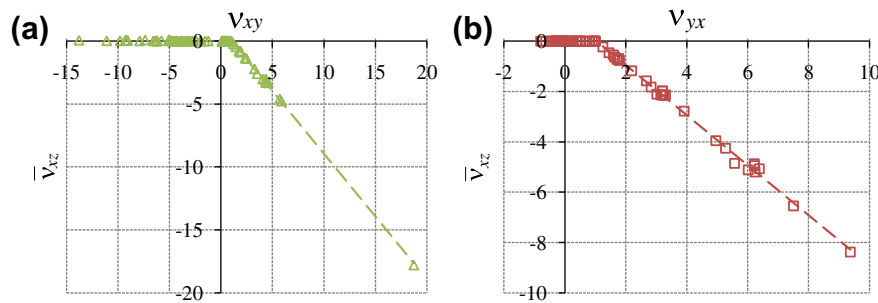

Fig. 5. Plots of the out-of-plane Poisson's ratio against the in-plane Poisson's ratio for all of the simulated systems for (a) loading in x, where the trend line for the cases with a negative out-of-plane Poisson's ratio is $y = -0.9996x + 1.0523$ with an $R^2 = 0.9991$ and (b) loading in y where the trend line is $y = -0.9958x + 1.0411$ with an $R^2 = 0.9969$ for cases having a negative out-of-plane Poisson's ratio.

This behaviour may be explained by referring to Fig. 2, which depicts a system built with a rubber like matrix ($\nu_M = 0.5$) being loaded in tension in the y-direction. In this case, as a result of loading, the honeycomb structure deforms through a flexure-like mechanism, resulting in the observed in-plane positive Poisson's ratio (in other words, the honeycomb structure shrinks in-plane).

As the composite is stretched, the ligaments in the structure flex and the soft matrix will be exposed to two different forces: a tensile force in the y-direction (which is the loading force) and a compressive force in the x-direction, as a result of the honeycomb framework moving in this direction. The tensile force experienced in the y-direction will tend to shrink the matrix in the z-direction,
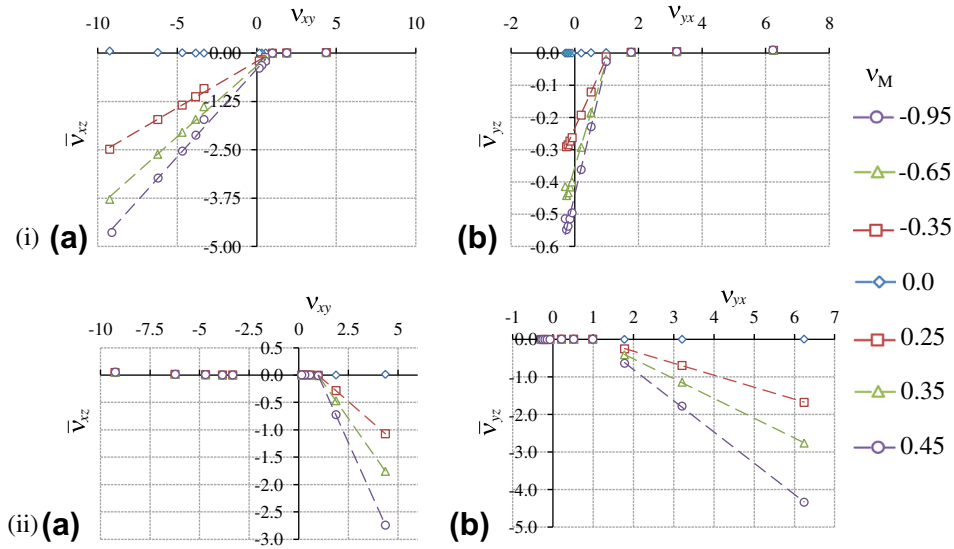
Fig. 6. Plots of the out-of-plane Poisson's ratio against the in-plane Poisson's ratio for systems with $l = 10$ cm, $h = 10$ cm, $t = 0.5$ cm, $z = 1$ cm and $\theta \in \{-25°, -20°, \ldots, -5°, 10°, 20°, \ldots, 60°\}$ (i), for $v_M \leqslant 0$ ($v_M \in \{-0.95, -0.65, -0.35, 0\}$) and (ii) for $v_M \geqslant 0$ ($v_M \in \{0, 0.25, 0.35, 0.45\}$) for (a) loading in x, (b) loading in y, showing the linear trend lines for each case, details of which are provided in the Supplementary information.

whilst the compressive force experienced in the x-direction will tend to expand the matrix out-of-plane, (due to the positive Poisson's ratio of the matrix, henceforth referred to as the 'Poisson's effect'). Here, it should be noted that the movement of the matrix as a result of the Poisson's effect occurs only along the third direction as this is the least hindered direction (movement in the x and y directions is restricted by the rigid honeycomb structure). This means that if the out-of-plane expansion of the matrix due to the compressive force is larger than its out-of-plane shrinkage due to the tensile loading force, the matrix will bulge out from the structure in the third direction, giving rise to the observed out-of-plane negative Poisson's ratio. Assuming that the matrix is isotropic, the composite will exhibit out-of-plane auxetic behaviour when the in-plane Poisson's ratio of the structure is higher than 1 as, in this case, the movement in the x-direction of the honeycomb structure will be larger than the movement in the y-direction (thus larger matrix compression than matrix expansion). For the other values of θ where the simulated in-plane Poisson's ratio is less than c. 1, the system exhibits an out-of-plane Poisson's ratio of approximately zero. The zero Poisson's ratio that arises in these cases may be explained by the fact that although the matrix gets thinner in the third direction, the honeycomb framework in the composite structure does not deform in this direction since it is much more rigid than the matrix, giving rise to the observed zero Poisson's ratio. Also worth noting is the fact that, referring to Fig. 5 and to the data presented in the Supplementary information, the out-of-plane Poisson's ratio seems to be independent of the thickness of the honeycomb in the third direction but is only dependent on the in-plane Poisson's ratio of the honeycomb structure.

This behaviour may be understood better if one considers a simple model of this system based on the assumptions that the matrix in which the honeycomb is embedded is isotropic, has a Young's modulus that is much lower than that of the honeycomb itself and the deformations throughout the matrix are uniform so that one can effectively consider an average out-of-plane strain, $\bar{\varepsilon}_z$ and Poisson's ratio $\bar{v}_{xz}$ and $\bar{v}_{yz}$. When the structure is subjected to stresses σ_x or σ_y acting in the x- or y-direction, the honeycomb framework deforms and experiences a strain in the x- and y-directions, ε_x and ε_y respectively. The matrix, which is much softer than the honeycomb, experiences these strains as well, together with an

additional strain $\bar{\varepsilon}_z$ in the z-direction due to the bulging out effect of the matrix from the pores. This effect is very pronounced for a matrix which has Poisson's ratio values that are close to 0.5 as can be found in rubber-like materials. The strains in the matrix can be related to each other through the generalised form of Hooke's law:

$$\bar{\sigma}_z(matrix) = c_{31}\varepsilon_x + c_{32}\varepsilon_y + c_{33}\bar{\varepsilon}_z \tag{1}$$

where c_{ij} are coefficients of the stiffness matrix of the matrix material. Since on deforming the structure, no stress is applied in the z-direction and the system is in equilibrium, $\bar{\sigma}_z = 0$ so that solving for $\bar{\varepsilon}_z$ in Eq. (1) yields an expression for the strain $\bar{\varepsilon}_z(matrix)$ of the matrix given by:

$$\bar{\varepsilon}_z(matrix) = -\frac{c_{31}\varepsilon_x + c_{32}\varepsilon_y}{c_{33}} \tag{2}$$

This strain in the z-direction is either approximately equal to zero (in cases where the matrix does not bulge out of the honeycomb, in which case the dimension in the third direction remains approximately constant at the value of the out-of plane thickness of the honeycomb) or is equal to that given by Eq. (2), i.e.

$$\bar{\varepsilon}_z = \max\left[0, -\frac{c_{31}\varepsilon_x + c_{32}\varepsilon_y}{c_{33}}\right] \tag{3}$$

For loading in the x-direction, realising that ε_x and ε_y for the matrix are the same as those for the honeycomb and the system, then, this equation can be rearranged and expressed in terms of the in-plane Poisson's ratio of the honeycomb v_{xy} as:

$$\bar{v}_{xz} = -\frac{\bar{\varepsilon}_z}{\varepsilon_x} = \min\left[0, \frac{c_{31} - c_{32}v_{xy}}{c_{33}}\right] \tag{4}$$

where $\bar{v}_{xz}$ is the out-of-plane Poisson's ratio of the matrix.

Furthermore, if one assumes that the matrix is also isotropic, then the terms c_{31}, c_{32} and c_{33} are given by:

$$c_{31} = c_{32} = \frac{E_M v_M}{(1 + v_M)(1 - 2v_M)} \tag{5}$$

$$c_{33} = \frac{E(1 - v_M)}{(1 + v_M)(1 - 2v_M)} \tag{6}$$

where v_M and E_M are the Poisson's ratio and Young's modulus of the matrix material.

In such case, Eq. (4) can be re-written as follows:

$$\bar{v}_{xz} = \min\left[0, \frac{c_{31}(1 - v_{xy})}{c_{33}}\right] = \min\left[0, \frac{v_M}{1 - v_M}(1 - v_{xy})\right] \tag{7}$$

Similarly, for loading in the y-direction,

$$\bar{v}_{yz} = \min\left[0, \frac{c_{31}(1 - v_{yx})}{c_{33}}\right] = \min\left[0, \frac{v_M}{1 - v_M}(1 - v_{yx})\right] \tag{8}$$

For rubber-like matrix materials which have a Poisson's ratio $v_M \approx 0.5$, Eqs. (7) and (8) reduce to

$$\bar{v}_{xz} = \min\left[0, 1 - v_{xy}\right] \tag{9}$$

$$\bar{v}_{yz} = \min\left[0, 1 - v_{yx}\right] \tag{10}$$

Note that Eqs. (7) and (8) clearly confirm that the out-of-plane Poisson's ratio of the composite material is highly dependent on the Poisson's ratio of the matrix. In fact, for a positive in-plane Poisson's ratio which is greater than 1, the higher the value of the matrix Poisson's ratio, the more negative the out-of-plane Poisson's ratio of the composite because of the higher tendency of the matrix to move out-of-plane. Likewise, for systems with a negative Poisson's ratio, the more auxetic the matrix is the more negative the out-of-plane Poisson's ratio for the composite. In the other cases where the matrix recedes inward on loading, the out-of-plane Poisson's ratio remains unaffected by the changes in the matrix Poisson's ratio as expected. Also, Eqs. (9) and (10) predict that for such a composite the variation of the out-of-plane Poisson's ratio with the in-plane Poisson's ratio should be linear with a gradient of −1. This trend is clearly reproduced in the simulations for the cases where the Poisson's ratio of the matrix is c. 0.5, the results of which are shown in Fig. 5.

The analytical model developed here to predict the out-of-plane Poisson's ratios of this composite system may be further extended with the help of the analytical models for the in-plane properties of hexagonal honeycombs developed elsewhere [2,3,74]. In fact, assuming that the in-plane Poisson's ratio of the honeycombs may be approximated by the expressions derived by Gibson et al. [2], which expressions are known to be very accurate for predicting the properties for honeycombs which deform primarily through flexure of the ribs, something which can be assumed if the matrix

is much softer than the honeycomb and the geometric dimensions of the honeycombs are appropriate, then expressions (9) and (10) above may be written in terms of the geometric parameters h, l and θ through:

$$v_{xy} = (v_{yx})^{-1} = \frac{\cos^2(\theta)}{(h/l + \sin(\theta))\sin(\theta)} \tag{11}$$

In cases when this cannot be assumed, for example, in cases where the in-plane thickness t is non-negligible, or for some values of θ which necessitate a correction factor, then it may be more appropriate to make use of the alternative expressions derived by Gibson and Ashby [3] and Masters and Evans [11] to predict the in-plane Poisson's ratio which is based on the assumption that the honeycombs deform through a combined flexure-hinging-stretching mechanism, and their extended versions derived by Grima et al. [74]. Based on the models by Gibson and Ashby [3], Masters and Evans [11] and Grima et al. [74], expression (11) above should be replaced by:

$$v_{xy} = \frac{\sin(\theta)\cos^2(\theta)[(l_{eff}/t)^2 + 1 + 2v_S]}{[h/l + \sin(\theta)]\left[(l_{eff}\sin(\theta)/t)^2 + 2(1 + v_S)\sin^2(\theta) + \cos^2(\theta)\right]} \tag{12}$$

$$v_{yx} = \frac{\sin(\theta)[h/l + \sin(\theta)]\left[1 + 2v_S + (l_{eff}/t)^2\right]}{\left[2h_{eff}/l_{eff} + \sin^2(\theta) + 2(1 + v_s)\cos^2(\theta) + (l_{eff}\cos(\theta)/t)^2\right]} \tag{13}$$

where h, l, θ and v_S have their usual meaning and h_{eff} and l_{eff} are respectively given by:

$$h_{eff} = h - \frac{t}{2}\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \tag{14}$$

$$l_{eff} = l - \frac{t}{2}\left[\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) + |\tan(\theta)|\right] \tag{15}$$

As evident from the plots in Fig. 7 which show the variation of the out-of-plane Poisson's ratio with the in-plane Poisson's ratio as obtained from the FE simulations and from the analytical models by Gibson et al. [2,3] and Grima et al. [74], the analytical models derived here provide an excellent description of the system, with
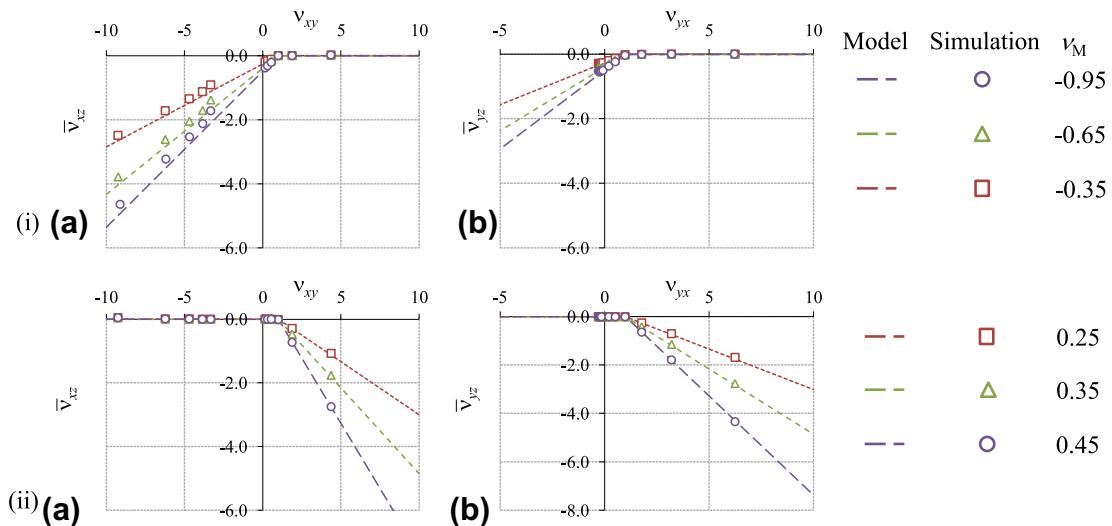

Fig. 7a. Plots of the out-of-plane Poisson's ratio against the in-plane Poisson's ratio for systems with $l = 10$ cm, $h = 10$ cm, $t = 0.5$ cm, $z = 1$ cm and $\theta \in \{-25°, -20°, \ldots, -5°, 10°, 20°, \ldots, 60°\}$ (i), for $v_M < 0$ ($v_M \in \{-0.95, -0.65, -0.35, 0\}$) and (ii) $v_M > 0$ ($v_M \in \{0, 0.25, 0.35, 0.45\}$) for (a) loading in x, (b) loading in y, together with plots of the analytical model, as developed in this paper, for a respective range of in-plane Poisson's ratios.
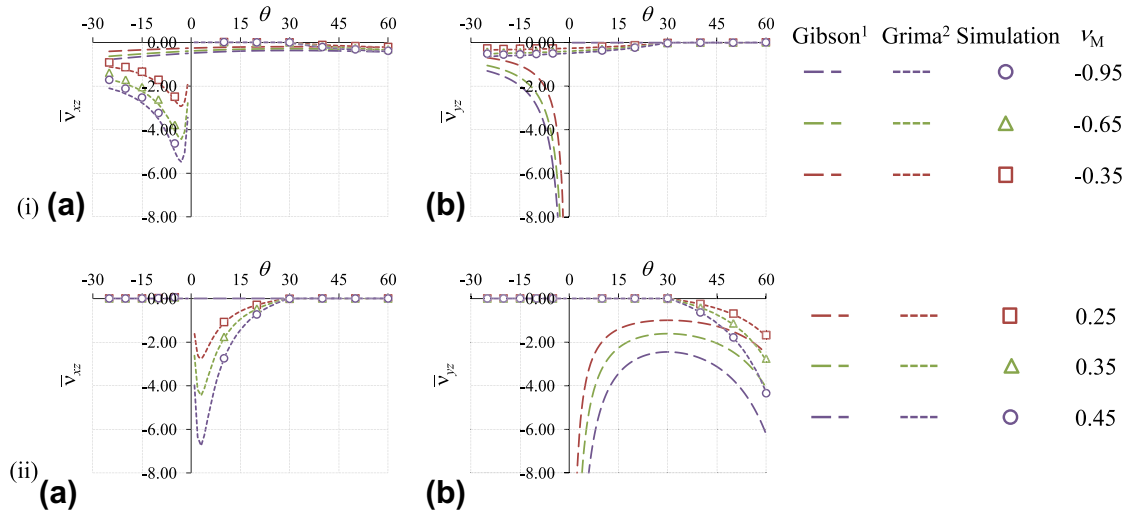
Fig. 7b. Plots of the out-of-plane Poisson's ratio against the in-plane Poisson's ratio for (i), for $v_M < 0$ and (ii) $v_M > 0$, for (a) loading in x, (b) loading in y, together with plots of the analytical model, as developed in this paper with the in-plane Poisson's ratios as derived from [1]the analytical model by Gibson et al. [2,3] and [2]Grima et al. [74].

the highly elegant model based on Gibson and Ashby's approach working for most of the systems modelled whilst the more complex model by Grima et al. [74] provides a good prediction in all of the cases considered. For example, for the non re-entrant honeycombs considered, the model proposed by Gibson gives a slight underestimation for v_{xy} this results in a less negative $\bar{v}_{xz}$ than that expected in the real systems or even predict $\bar{v}_{xz}$ to be zero when in fact the sys-

tem would be auxetic (This would happen if the underestimation is very significant such that v_{xy} predicted by the equations based on Gibson's et al. model is lower than 1.) Similarly, for loading in the y-direction, Gibson's model gives a slight overestimation for v_{yx}. This also results in an overestimation of the out-of-plane auxeticity and in some cases, it may even predict auxetic behaviour when not the case.
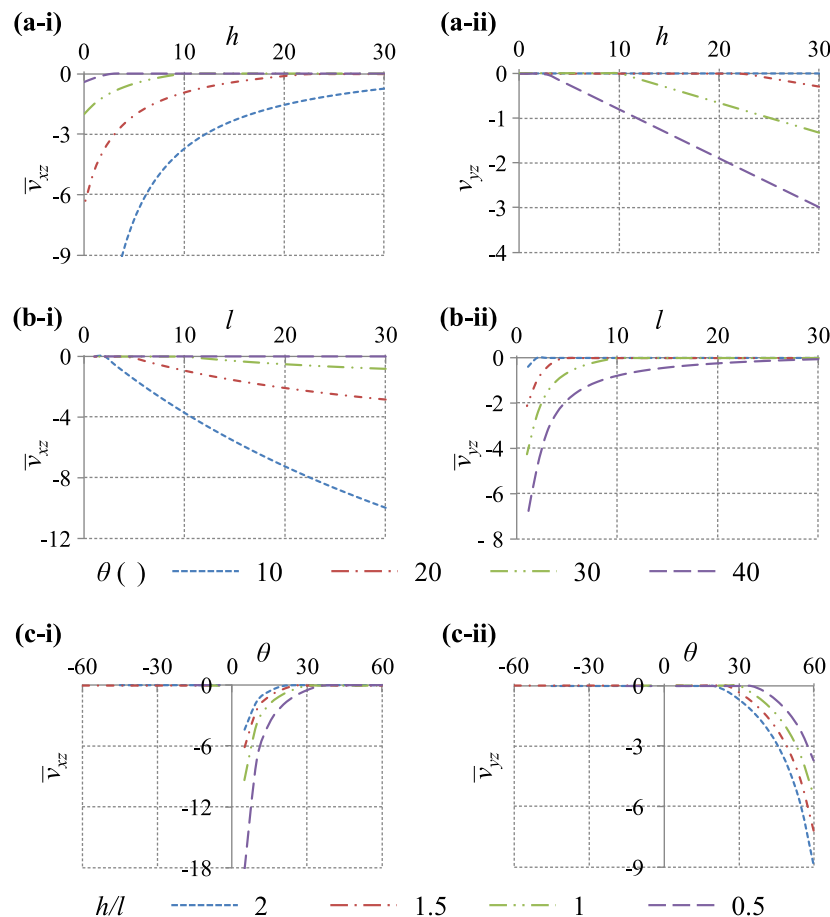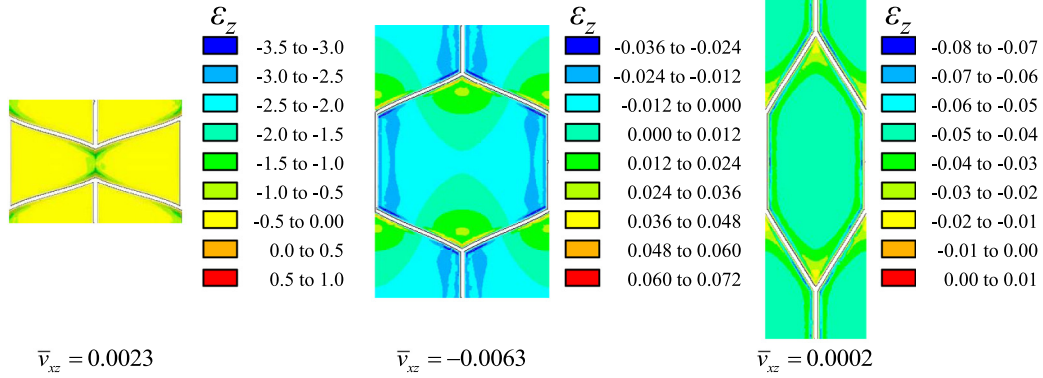

Fig. 8. Plots showing how (i) v_{xz} and (ii) v_{yz} vary with (a) the length of the vertical ribs, h ($l = 10$ cm) (b) the length of the inclined ribs l ($h = 10$ cm) and (c) the angle θ, as predicted by the analytical model derived here using equations for v_{xy} and v_{yx} derived by Grima et al. [74]. In all cases, the thickness $t = 0.1$ cm.
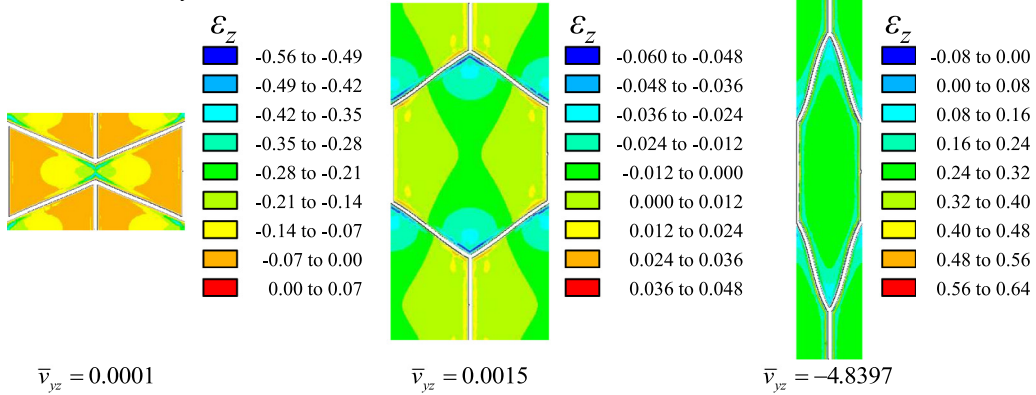
0.05 strain in the x-direction

Fig. 9. Contour plots for the out-of-plane strain, ε_z (=0.05) for loading (a) in the x- and (b) y-directions for systems having a θ-value of (i) $-25°$, (ii) $30°$ and (iii) $60°$, showing that places of maximum displacement tend to be offset from the centre. In all cases $l = h = 10$ cm, $t = 0.5$ cm and $z = 1$ cm, while $v_M = 0.49$).
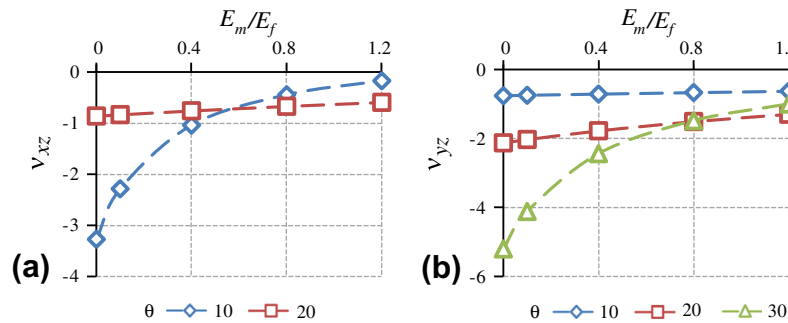

Fig. 10. Plots of (a) v_{xz} (b) v_{xy} against ratios of the Young's moduli of the matrix to the Young's modulus of the empty honeycomb framework for the respective direction of loading. Shown are angles for which there is a negative out-of-plane Poisson's ratio for the particular direction of loading.

More importantly, expressions (7), (8), (11)–(14), (and) (15) also provide a tool to help experimentalists design composites with specific tailor-made properties. For example, these expressions predict that in cases where the matrix has a positive Poisson's ratio (i.e. $v_M/(1 - v_M)$ is positive) for maximising the out-of-plane auxetic effect for loading in the x-direction, the in-plane Poisson's ratio v_{xy} of the honeycomb should ideally be much greater than 1. Theoretically this can be done by maximising the numerator and minimising the denominator in Eq. (12). According to this equation, the simplest way to achieve this is by decreasing h since this would effectively make the denominator smaller whilst leaving the numerator constant. Thus, the ideal structure for out-of-plane auxeticity when loading in the x-direction would be one where $h = 0$, i.e. when the honeycomb resembles a wine-rack structure. On the other hand, control of the out-of-plane Poisson's ratio through modification of l and θ is less straightforward since these

parameters contribute to both the numerator and denominator. However, analysis of these two terms suggests that unless l and θ are very small, an increase in l or a decrease in θ results in an increase in the out-of-plane Poisson's ratio, as is evident from Figs. 6–8bi and ci. Similar arguments can also be put forward when the composite is loaded in the y-direction. In this case, however, an increase in h, a decrease in l and an increase in θ are desirable for higher out-of-plane auxeticity. It is important to note that although the equations seem to point to extreme values for maximum out-of-plane auxeticity, one should keep in mind that other properties like the in-plane Young's modulus, which are also affected by changes in these parameters, are not compromised.

Before we conclude, it should be emphasised that the work presented here has made various assumptions and considered only a subset of the possible systems which can be constructed in the manner proposed here. For example, this system only considered

the classical hexagonal honeycombs even if such properties could probably be obtained from other cellular systems. Also, the derivation presented in this work assumed that the matrix is much softer than the honeycomb and that it bulges out in a uniform manner as if it was unbound at the matrix-honeycomb interface. In reality, as the plots for the out-of-plane strain component for the deformed structures in Fig. 9 show, the deformation of the matrix is not uniform. Although these plots at first suggest that deformation of the matrix occurs in such a way that the out-of-plane displacement is at a maximum at the centre of the pore and at a minimum in the vicinity of the ligaments to which they are bound, the actual deformation is much more complex. In particular, if one focuses towards those regions of highest out-of-plane deformation, it becomes apparent that in fact, these regions, albeit small, may be off-centred. However, since the z dimension recorded in the simulation is the average one, then the derivation gives equivalent results to that of the measurements from the simulations. One may argue that this averaging approach is an over simplification of the real scenario, but here it should be noted that this corresponds to a modest estimation of the out-of plane auxeticity. In fact, one should keep in mind that averaging out of the nodal displacements may lead to some underestimations of auxeticity since regions of large out-of-plane displacements arising from non-uniform deformations of the matrix may in reality result in a higher auxeticity than that predicted. Nevertheless, a detailed analysis of the systems modelled suggests that Poisson's ratios estimated from the maximum displacements are not too dissimilar to those reported here and the same trends are observed.

Furthermore, it is important to emphasise that what is presented here is just a preliminary modelling study which needs to be extended to attain a full picture of the mechanical behaviour of the proposed systems. Such further studies are not limited to experimental work which will obviously provide the definite proof that what is being proposed here indeed works, but also further modelling and simulations based studies. For example, in the simulations and model reported here, it was always assumed that the modulus of the matrix is much lower than that of the honeycomb, in order to enhance the auxetic effect of such systems to provide a proof of concept. However, materials with such properties may not be easy to obtain and thus it would be interesting to analyse to what extent the relative stiffness of the matrix affects the out-of-plane auxeticity. Preliminary simulations, similar to the ones in another study by Evans et al. [32], the results of which are shown in Fig. 10, clearly show that as the stiffness of the matrix increases, the out-of-plane auxetic effect also decreases. Also all simulations performed here reported the results at a fixed constant strain. It is well known that the mechanical properties of honeycombs, in particular their Poisson's ratios, are highly strain dependent and it would be interesting if profiles of the out-of-plane Poisson's ratios against strain are obtained through further simulations. Also, in this work it was assumed that the systems modelled here were constructed in an ideal defect-less manner, whilst in reality, such conditions of ideality may be difficult to achieve.

However, the main strength of this work remains its relative simplicity. It should be emphasised that the systems presented which are supposed to exhibit auxetic behaviour, are constructible using readily available materials. This means that the cost of production of such auxetic composites may be more reasonable than that for the manufacture of some previously proposed auxetics.

5. Conclusion

This work has proposed a novel concept which permits the production of auxetic composites made from readily available materials and components, such as metal honeycombs embedded in a soft rubbery matrix. It was shown that the out-of-plane auxeticity of the composite is dependent on the in-plane properties of the honeycomb and is also affected by changes in the Poisson's ratio of the matrix. Given the many benefits which may be attributed to auxetic systems such as the ones proposed here, and the estimated reduced production costs to make these composites, it is hoped that this work would encourage experimentalists and industrialists to consider the manufacture of the proposed systems even at a commercial scale.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.compstruct.2013.06.009.

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