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On the Effect of the Mode of Connection between the Node and the Ligaments in Anti-Tetrachiral Systems**

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Chiral systems may exhibit auxetic behavior, i.e. they may have a negative Poisson's ratio. This particular property has led to their being studied extensively by several authors. A Finite Elements Study is presented here, investigating the mode of connection between the nodes and ligaments in the anti-tetrachiral structure. The results show that the amount of gluing material used to attach the ligaments to the node will not affect the Poisson's ratio, but may have a large influence on the observed stiffness of the structure (Young's modulus). It is also shown that the stiffness of the glue will have a large effect on the mode of deformation of the chiral system. This change in mechanism was found to effect the stiffness of the structure but not its Poisson's ratios.

1. Introduction

Materials with a negative Poisson's ratio, commonly called auxetics, have the unusual property of expanding laterally when a uniaxial load is applied.[1–3] This behavior is the consequence of the materials having a particular geometry and deformation mechanism. Importantly, auxetic behavior has been shown to be scale-independent, meaning that these mechanisms can be found operating at the macro-, micro-, and nanoscales.[2,4]

Chiral systems, which are perfect examples of auxetic systems, were first proposed by Wojciechowski,[3] implemented as a structure by Lakes,[2] and later Sigmund,[5] and have since been the subject of numerous detailed studies[6–28] due to their numerous potential applications, which include satellite antenna designs,[29,30] sensors,[31] and stent geometries.[32]

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The Young's moduli and Poisson's ratios of the generalized anti-tetrachiral system have been derived and discussed in the past.[33] It has also been shown that the on-axis properties of such systems could be fine-tuned to meet desired requirements.[33] However, the majority of past derivations, with some notable exceptions,[34] are unfortunately limited to describe only highly idealized systems. For instance, they do not take into account the area of the ligament needed to glue with the node, or the properties of the glue itself, even though it has been shown elsewhere that, if necessary, a flush tangent attachment of the ligament with external glue can be avoided.[31] The present paper attempts to rectify these problems by studying the effects of the amount and location of the gluing material, together with its mechanical properties, on the mechanical properties of the general anti-tetrachiral system.

2. Experimental Section

The analytical model derived previously[33] had assumed that the ligament just touches the node. This scenario, which is highly unrealistic, is depicted in Figure 1a. In reality, a situation similar to that depicted in Figure 1b–d would be expected, where gluing material fills the space between the ligament and node.

This limitation of the analytical model can be removed by investigating the effect of the gluing material on the mechanical properties of the generalized flexing anti-tetrachiral systems. Two approaches were made: an analytical model and FEA simulations. In the analytical approach, expressions, which predict the Poisson's ratio and Young's moduli in terms of the different lengths, thicknesses and stiffness of the ligaments, and the amount and location of gluing material between the ligament and node, were derived. The analytical model was then validated through FEA simulations, which have the advantage

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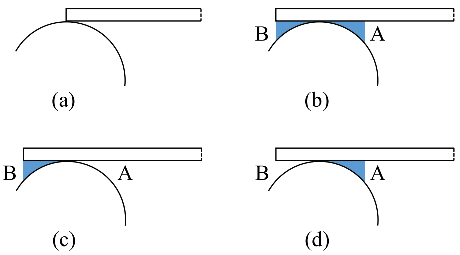

Fig. 1. (a) A magnification of the node showing a highly unrealistic situation where the ligament and node only touch at one point, (b) a depiction of a more realistic system where glue is applied to both sides of the node, (c) a depiction where glue is applied only on side B of the node, and (d) a depiction where glue is applied only on side A of the node.

of being more realistic, but the disadvantage that they can only predict the properties for particular systems.

2.1. Analytical Approach

The system shown in Figure 2 will be considered. The nodes in this figure have a radius r and are made of a material having Young's modulus E_s. The ligaments in the Ox_1 direction have a length l_1, a thickness t_1 and are made of a material having a Young's modulus of E_{s1}. Similarly, the ligaments in the Ox_2 direction have a length l_2, a thickness of t_2 and are made up of a material having a Young's modulus of E_{s2}. The geometric parameters in the Ox_1 directions are allowed to have different geometric and mechanical properties than those aligned in the Ox_2 direction, as discussed by Gatt *et al.*,[33] namely:

- different lengths l_1 and l_2,
- different thicknesses t_1 and t_2,
- different Young's moduli E_{s1} and E_{s2} of the constituent material.

Two additional variables, g_{I} and g_{O}, represent the length of the ligament which is used for gluing the ligaments to the node. More specifically, referring to Figure 2b, g_{I} represents the amount of gluing material that is on the side of the ligament which is connected to another node, while g_{O} represents the amount of gluing material that is on the part of the ligament which terminates at the current node.

In the derivation, it shall be assumed that all deformations occur as a result of flexure of the ligaments. In particular:

- the lengths of the ligaments remain constant (i.e. the ligaments do not stretch),
- the nodes remain perfectly rigid and simply rotate by an angle ϕ which is equal to the angular deflection at the ends of the ligaments,
- the line joining the center of the nodes with the point of contact between the ligaments and the nodes remain at 90° to the ligaments (i.e. the point of contact between the ligaments and nodes is not acting as a hinge).

The analytical model will therefore only capture deformations, which occur as a result of "flexure" of the ligaments. Additionally, it shall also be assumed that:

- the point of contacts between the nodes and the ligaments is always at a distance g_{O} from the end of the ligament, and that the total ligament length is equal to $l_i + 2g_{\mathrm{O}}$,
- the glue has mechanical properties which are comparable to that of the node (i.e. the glue is assumed to be perfectly rigid),
- energy will only be stored in the portion of the ligaments which are not connected with the nodes through the glue (i.e. the energy in the system will be stored only in the portion of the ligaments that are in between of the nodes and that are free to move, or l_i (2g_{I}, which will henceforth be referred to as $l_{i,\mathrm{eff}}$ the effective length of the ligaments in the Ox_i directions). This is represented by the unhighlighted regions in Figure 2c.

The unit cell as shown in Figure 2 has projections in the Ox_i directions (where $i = 1,2,3$) given by:

$$
\begin{aligned}
X_1 &= 2(l_{1,\mathrm{eff}} + 2g_{\mathrm{I}}) = 2l_1 \\
X_2 &= 2(l_{2,\mathrm{eff}} + 2g_{\mathrm{I}}) = 2l_2 \\
X_3 &= z
\end{aligned}
\tag{1}
$$

where z is the thickness in the third direction.

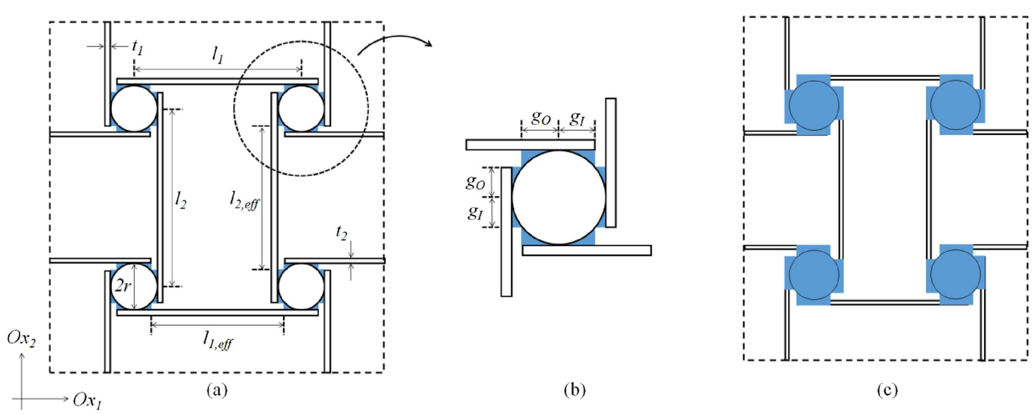

Fig. 2. (a) The anti-tetrachiral unit cell, showing how the ligaments are glued to the nodes and (b) a representation of the gluing variables g_{I} and g_{O}. (c) During deformation, the parts of the anti-tetrachiral system highlighted in blue are considered to be rigid, implying that the energy is stored in the unhighlighted parts.

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2.1.1. The In-Plane Poisson's Ratio for Loading On-Axis

The Poisson's ratio (ν_{ij}) in the Ox_1–Ox_2 plane for loading in the Ox_i may be defined as:

$$\nu_{ij} = -\frac{\epsilon_j}{\epsilon_i} = \frac{X_i^{\text{init}}\Delta X_j}{X_j^{\text{init}}\Delta X_i} \tag{2}$$

where X_i^{init} are the projections of the undeformed unit cell in the Ox_i directions and ΔX_i are small changes in length in the X_i^{init}.

Also, in this case, on-axis uniaxial loads σ_i in the Ox_i direction will result in flexure of the ligaments, which will cause a rotation of the nodes by an angle ϕ, and thus the distance between the centers of the nodes will change from l_i to:

$$l_i + 2\left(r + \frac{t_i}{2}\right)\sin(\phi) \approx l_i + 2\left(r + \frac{t_i}{2}\right)\phi \tag{3}$$

Therefore the, strain generated by the applied stress would be equal to:

$$\epsilon_i = \frac{\Delta X_i}{X_i^{\text{init}}} = \frac{2(2e)}{2(l_i)} = \frac{2\left(r + \frac{t_i}{2}\right)\phi}{l_i} = \frac{(2r + t_i)\phi}{l_i} \tag{4}$$

Hence, the Poisson's ratio would be equal to

$$\nu_{ij} = -\frac{\epsilon_j}{\epsilon_i} = -\frac{(2r + t_j)\phi}{l_j} \cdot \frac{l_i}{(2r + t_i)\phi} = -\frac{(2r + t_j)l_i}{(2r + t_i)l_j} \tag{5}$$

Note that, this expression is the same as the one derived by Gatt *et al.*[33] where the effect of glue was not included, a property which suggests that the amount of glue is not expected to affect the Poisson's ratio of the system.

2.1.2. The In-Plane Young's Modulus for Loading On-Axis

The Young's modulus in the Ox_i direction may be derived through the conservation of energy approach. Recall that for this "idealized flexure model" it being is assumed that the model deforms through pure "flexure" of the ligaments. This implies that, referring to Figure 3, all the strain energy will be stored as "bending energy" in the unhighlighted portions of the ligaments, which have effective lengths of $l_{i,\text{eff}}$. It must also be emphasized that the energy stored in the ligaments lying in the Ox_1 direction may be different from the energy stored in ligaments in the Ox_2 direction.

The moment M acting on each end of the ligament with an effective length $l_{i,\text{eff}}$ due to an applied stress in the Ox_i direction, can be related to the rotation of the nodes ϕ, through standard beam theory[35]:

$$\phi = \frac{Ml_{i,\text{eff}}}{2E_{si}I_i} \tag{6}$$

where E_{si} is the intrinsic Young's modulus of the material of the ligament i whilst I_i is the cross-section's second moment of area of the ligament i in the Ox_i direction. Since the cross-sections are taken to be rectangular, having a thickness of t_i

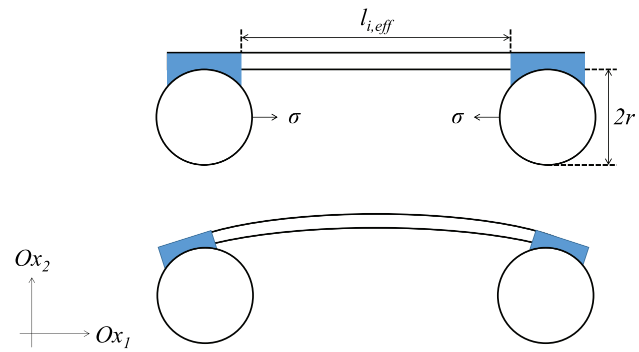

Fig. 3. A system made up of two nodes and one ligament when subjected to mechanical stress.

and width of z, then:

$$I_i = \frac{t_i^3 z}{12} \tag{7}$$

Equation 6 can be re-written in terms of the moment as:

$$M = \frac{zE_{si}t_i^3}{6l_{i,\text{eff}}}\phi = \frac{zE_{si}t_i^3}{6(l_i - 2g_{\text{I}})}\phi \tag{8}$$

Using the method described previously,[33] the Young's Modulus may be calculated by relating the strain energy per unit volume with the work done as follows:

$$E_i^{\text{anti-tet sqr}} = \frac{2W_{\text{T}}}{V_{\text{cell}}\epsilon^2} \tag{9}$$

where W_{T} is the total strain energy that accompanies a strain ϵ and V_{cell} is the volume of the unit cell which is given by:

$$V_{\text{cell}} = 4l_1 l_2 z \tag{10}$$

The energy W_i stored in each flexed ligament i may be obtained from:

$$W_i = 2\int_0^{\phi} M_{\phi}\,\mathrm{d}\phi \tag{11}$$

Thus, from Equation 9 and 11, the total energy per unit cell is equal to:

$$\begin{aligned} W_{\text{T}} = 4(W_{\text{Lig1}} + W_{\text{Lig2}}) &= 4z\left(\frac{E_{s1}t_1^3}{6l_{1,\text{eff}}} + \frac{E_{s2}t_2^3}{6l_{2,\text{eff}}}\right)\phi^2 \\ &= 4z\left(\frac{E_{s1}t_1^3}{6(l_1 - 2g_{\text{I}})} + \frac{E_{s2}t_2^3}{6(l_2 - 2g_{\text{I}})}\right)\phi^2 \end{aligned} \tag{12}$$

Therefore, by substituting Equation 4, 10, and 12 in Equation 9, the Young's modulus of the generalized flexing anti-tetrachiral with square nodes may be given by:

$$\begin{aligned} E_i^{\text{anti-tet glue}} &= \frac{l_i}{3(2r + t_i)^2 l_j}\left(\frac{E_{s1}t_1^3}{l_{1,\text{eff}}} + \frac{E_{s2}t_2^3}{l_{2,\text{eff}}}\right) \\ &= \frac{l_i}{3(2r + t_i)^2 l_j}\left(\frac{E_{s1}t_1^3}{(l_1 - 2g_{\text{I}})} + \frac{E_{s2}t_2^3}{(l_2 - 2g_{\text{I}})}\right) \end{aligned} \tag{13}$$

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which in the special case that the properties and dimensions for the horizontal ligaments are the same as the ones in the vertical ligaments, reduces to:

$$E_i^{\text{anti-tet glue}} = \frac{E_{si}t^3}{3(r+t)^2}\frac{1}{l_{i,\text{eff}}} = \frac{E_{si}t^3}{3(r+t)^2}\frac{1}{(l_i - 2g_\text{I})} \quad (14)$$

2.2. FEA Approach

FE simulations were carried out in order to examine the validity of the results of the analytical derivation, and in turn those of the initial assumptions made. The simulations were designed to test the following three assumptions:

- the Young's moduli are truly independent of g_O (i.e. no energy will be stored in the ligament part that is spanning away from the node);
- the expressions derived above (Eq. 5 and 14) give a true description of the system;
- the properties of the glue affect the properties of the whole system.

In line with previous work,[19,33] four dimensional parameters, α_1, α_2, β_1, and β_2, were defined, where $\alpha_1 = l_1/r$, $\alpha_2 = l_2/r$, $\beta_1 = t_1/r$, and $\beta_2 = t_2/r$. Given the symmetry of the structure and the loading conditions, the system illustrated in Figure 2 was constructed using the commercially available finite elements code ANSYS.[36] The system was represented through a quarter unit cell and given the boundary conditions shown in Figure 4 to produce a representation of an infinite system which is analogous to the analytical model. Both the nodes and ligaments were meshed using Plane 82 elements, a 2D 8-Node Structural Solid, with plain strain capability.[37] Convergence tests was performed in order to choose an appropriate element size for the systems modeled.[38] The mesh density in this case was calculated using the ANSYS Smart Size Feature with the size level set to 1, and all the systems were solved linearly. Note that the models were subjected to strains in the Ox_2 direction in all simulations. The constituting materials of the ligaments and nodes were modelled using the mechanical properties of typical PVC plastics.[39]

2.2.1. Testing the Influence of g_O and g_I

In order to test whether the Young's moduli are truly independent of g_O (i.e. no energy will be stored in the ligament part that is spanning away from the node) three sets of simulations were performed on a system of dimensions $\alpha_1 = \alpha_2 = 5$, $\beta_1 = \beta_2 = 0.1$, $E_{s1} = E_{s2} = 3$ GPa, and $E_{sn} = 4$ GPa. g_O and g_I were varied as follows:

- Set I: A system with $g_\text{I} = 0$ cm and $g_\text{O} = 0.1$ cm was compared to a system with $g_\text{I} = 0$ cm and $g_\text{O} = 0.75$ cm.
- Set II: A system with $g_\text{I} = 0.1$ cm and $g_\text{O} = 0$ cm was compared to a system with $g_\text{I} = 0.75$ cm and $g_\text{O} = 0$ cm.
- Set III: A system with $g_\text{I} = 0.75$ cm and $g_\text{O} = 0$ cm was compared to a system with $g_\text{I} = 0.75$ cm and $g_\text{O} = 0.75$ cm.

Each of these systems was simulated as described previously.[33] Each system was subjected to a strain of 5%, using the boundary conditions described in Figure 4. Note that even though in these simulations the systems were subjected to strains in the Y-direction, the same conclusions would be obtained if the system had been load in the Ox_1 direction as a result of the symmetry of the anti-tetrachiral systems.

2.2.2. Testing the Results of the Analytical Model

Six sets of simulations was carried out to test whether Equation 13 and 14 give a true description of the anti-tetrachiral system. The sets were based on systems having the geometric parameters and intrinsic mechanical properties given in Table 1. For each of the three simulation conditions in

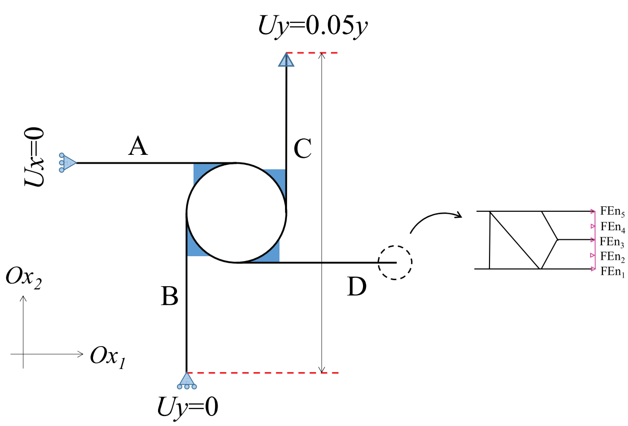

Fig. 4. The boundary conditions which were applied to the model constructed in ANSYS to represent an infinite system, where the end of ligament A was constrained not to move in the X-direction, the end of ligament B was constrained not to move in the Y-direction, the FEA nodes at the end of ligament D were constrained to have the same displacement in the X-direction, whilst a strain of 5% was applied on the end of ligament C.

Table 1. The parameters, which were varied during the simulations.

Set	1	2	3	4	5	6
α_1	5, 10, 15	10	10	10	10	10
α_2	10	5, 10, 15	10	10	10	10
β_1	0.1	0.1	0.05, 0.1, 0.15	0.1	0.1	0.1
β_2	0.1	0.1	0.1	0.05, 0.1, 0.15	0.1	0.1
Es_1	3	3	3	3	2, 3, 4	3
Es_2	3	3	3	3	3	2, 3, 4
Es	4	4	4	4	4	4

For each experiment, g_I was varied within between 0.10 *r* and 0.50 r (included) with increments of 0.10 r.

each set, g_I was varied between $0.10\,r$ and $0.50\,r$ (included) with increments of $0.10\,r$.

As before, each system was subjected to a strain of 5%, using the boundary conditions described in Figure 4.

2.2.3. *Determining Whether the Properties of the Glue Influence the Properties of the Whole System*

In the simulations described so far, the important assumption that the glue used had comparable mechanical properties to the properties of the nodes of the systems was made. In reality, this assumption may not necessarily be valid, and the glue properties might have significant effects on the observed properties of the tetrachiral systems. To test the assumption a set of simulations was performed on the system described earlier. The dimensions used were $\alpha_1 = \alpha_2 = 10\,\text{cm}$, $\beta_1 = \beta_2 = 0.1\,\text{cm}$, $E_{s1} = E_{s2} = 3\,\text{GPa}$, and $E_{sn} = 4\,\text{GPa}$. The Young's modulus of the glue $g_I = 0.5\,\text{cm}$ was varied within the range 4×10^3, 4×10^4, 4×10^5, …, 4×10^{12} Pa.

As before, each system was subjected to a strain of 5%, using the boundary conditions described in Figure 4.

3. *Results and Discussion*

A qualitative analysis of the deformed structures, as simulated through the FE simulations, suggests that the addition of glue to the anti-tetrachiral system does not affect its mode of deformation. In all cases, as the systems are pulled in the *Y*-direction, there is flexure of the vertical ligaments which causes a rotation of the nodes, which in turn causes flexure of the horizontal ligaments hence providing the observed negative Poisson's ratio effect.

The results, discussed in detail below, show that:

- the glue g_O does not influence the properties of the anti-tetrachiral system,
- in most cases, there is good agreement between the predictions made by the simple analytical model (Eq. 14) and the FEA simulation results,
- the ratio of the Young's modulus of the glue to the Young's modulus of the ligaments changes the deformation mechanism and Young's modulus of the system, but not its Poisson's ratio.

3.1. *Testing the Influence of g_O and g_I*

When structures having the same parameters of $\alpha_i = 5\,\text{cm}$, $\beta_i = 0.1\,\text{cm}$, and $E_{si} = 3\,\text{GPa}$ but different values of g_O and g_I were each subjected to the same strain of 5% in the *Y*-direction, they all extended in the *X*-direction by the same amount (5% since the chirals were regular, indicating the same Poisson's ratios of −1) but had different deformation profile which depended on the amount of glue g_I. On the other hand the deformation was unaffected by the amount of glue g_O. These results are illustrated in Figure 5. More specifically, systems, which had the same amount of glue g_I but different amounts of glue g_O deformed in exactly the same manner when subjected to the same amount of uniaxial strain. The flexed ligaments adopted an identical curvature while the nodes rotated by the same amount. Inversely, systems having the same amount of glue g_O but different amounts of glue g_I deformed in a different manner by adopting different ligament curvatures, even though they retained the same Poisson's ratios. This also indicates that the Young's moduli and Poisson's ratios of these systems were independent of g_O. However, systems having different amounts of glue g_I deformed in different manners when subjected to the same amount of stress, indicating that the Young's moduli were different. All this is in accordance with the analytical model.

3.2. *Testing the Results of the Analytical Model*

As shown in Figure 6, only minimal deformations in the shape of node and the parts of the ligaments, which are glued to them were noted in the deformed systems. This is in accordance with the assumption made in deriving the analytical model that the portion of the structure, which is highlighted in Figure 2c was assumed to behave like a rigid

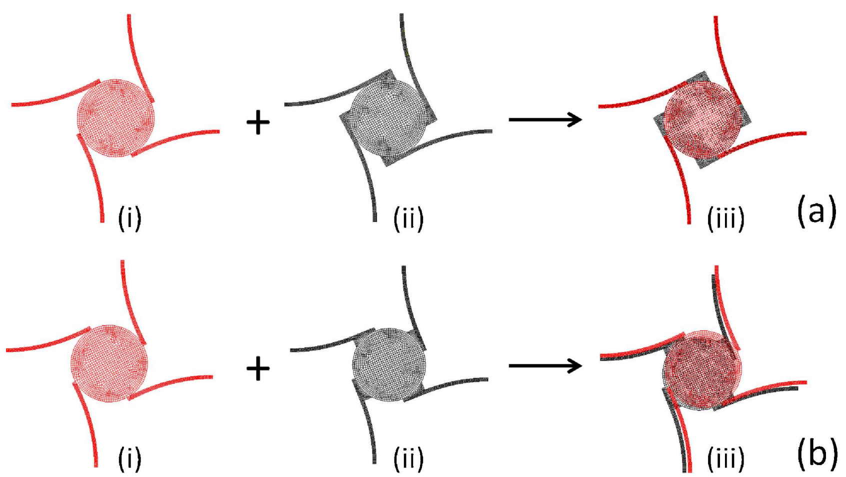

Fig. 5. Figures showing the modeled anti-tetrachiral system where systems with (a) (i) $g_I = 0$ cm and $g_O = 0.1$ cm, and (ii) $g_I = 0$ cm and $g_O = 0.75$ cm, were superimposed (iii); (b) (i) $g_I = 0.1$ cm and $g_O = 0$ cm and (ii) $g_I = 0.75$ cm and $g_O = 0$ cm, were superimposed (iii). A strain of 5% has been applied in all cases.

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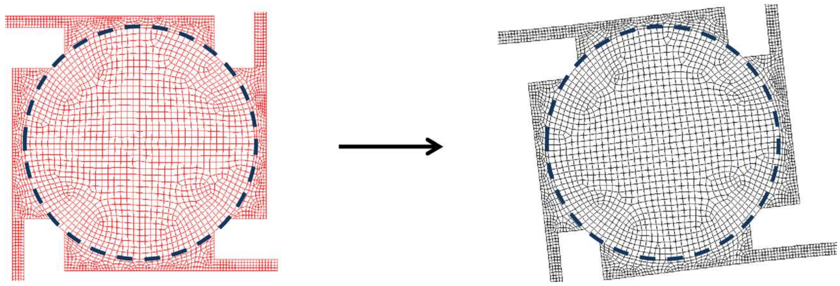

Fig. 6. An undeformed (left) and deformed (right) part of the anti-tetrachiral system showing that upon 5% strain the node and glue remain rigid.

unit when compared to the portion of the ligaments which are not glued to the node.

This is further confirmed by the plots of the Poisson's ratios and Young's moduli as predicted by the analytical model and the FE simulations for the systems in Table 1. Such plots, which are given in Figure 7, confirm that in most cases there is good agreement between the predictions made by the simple analytical model derived in Section 2.1 and the detailed FE simulations performed in Section 2.2. Moreover, these plots also highlight the relationship between the Poisson's ratios and Young's moduli with the amount and position of the gluing material used.

The results obtained clearly indicate that the Poisson's ratio of the system is not affected by the amount of gluing material used, but only by the ratio of the relative ligament lengths and thicknesses, in accordance with previous studies.[33] Furthermore, the results indicate that the Young's moduli of the general flexing anti-tetrachiral system are not only dependent on the ratio of the ligament lengths, ligament thicknesses, and Young's moduli of the ligament material as discussed by Gatt *et al.*[33] but also dependent on the amount of glue used, as well as its position. While varying the amount of gluing material g_O will have no effect on the Young's moduli of these systems, an increase in the amount of gluing material g_I will result in an increase in the Young's modulus of the general flexing anti-tetrachiral.

The same observations can also be explained by analyzing the energy stored in the ligaments when stress is applied. In the first instance, as g_O is increased, the length of the ligament which deforms is not changed. Since such systems will have the same ligament curvatures, the energy stored in the ligaments will be the same irrespective of the amount of glue g_O. On the other hand, if g_I is increased, the length of ligament between the nodes will effectively become shorter, which implies that they become more difficult to bend and therefore the Young's modulus of the structure will increase. This particular effect of glue g_I can easily be inferred through Equation 13. In fact, as g_I becomes larger, the term $l_i{-}g_I$ becomes smaller, so consequentially the Young's modulus becomes larger.

This also implies that the larger the ligament lengths are relative to the amount of gluing material used (g_I), the smaller an effect g_I has on the Young's modulus of the generalized anti-tetrachiral system. Consequently, the shorter the ligament becomes relative to the amount of gluing material (g_I) used, the greater the effect of the glue on the Young's modulus of these systems. This behavior may be directly inferred from Equation 13, which has the terms $l_i{-}2g_I$ in its denominator since as l_i increases, $l_i{-}2g_I$ approaches l_i. This behavior can also be illustrated by considering a particular situation with a system having nodes with radius of 5 and 2.5 cm of gluing material g_I. If one had to compare two such systems, where in the first case the ligament is 15 cm long, while in the second case the ligament is 50 cm long, then the part of the ligament, which flexes in the first case is 15 cm $\times$ (2.5 cm $\times$ 2) = 10 cm, which is 33% less than the original length, while when considering the second case, the part of the ligament that flexes is 50 cm $\times$ (2.5 cm $\times$ 2) = 45 cm, which is only 10% less than the original length.

3.3. Determining Whether the Properties of the Glue Influence the Properties of the Whole System

The effects of varying the glue strength on the deformation mechanism of the anti-tetrachiral system are shown in Figure 8. In Figure 8a, the glue is much softer than the nodes, while in Figure 8c the glue is much harder than the node. These systems are compared to the deformations of the anti-tetrachiral systems where the properties of the glue are the same as those of the node, which corresponds to the system constructed in Section 2.2. Plots of the observed Poisson's ratios and Young's moduli of these systems are shown in Figure 9. An analysis of the deformed shapes clearly illustrates that the behavior of systems where the glue is much softer when compared to the rest of the structure is significantly different from those where the glue is hard. Although in both systems there is node rotation, the system with soft glue deforms primarily through a hinging type mechanism, where the ligaments and the nodes behave as rigid units and the glue acting as the hinge. Such systems may obviously be considered as hinging systems, and may be shown to be equivalent to the systems considered in detail by Grima.[40] In contrast to this, the systems where the glue is hard behave through the ligament flexure mechanism in accordance to what is described in Section 2.

In real systems, what is likely to happen is that both the hinging mechanism and the flexure mechanism operate concurrently, the relative extent depending on the properties of the glue. For example, if standard plastic PVC is used for the manufacture of the slender ligaments and nodes, and the system is connected with a rubbery like glue (e.g. a soft silicone glue), then it is likely that the deformation mechanism will be predominantly hinging. In contrast, if an epoxy resin type glue is

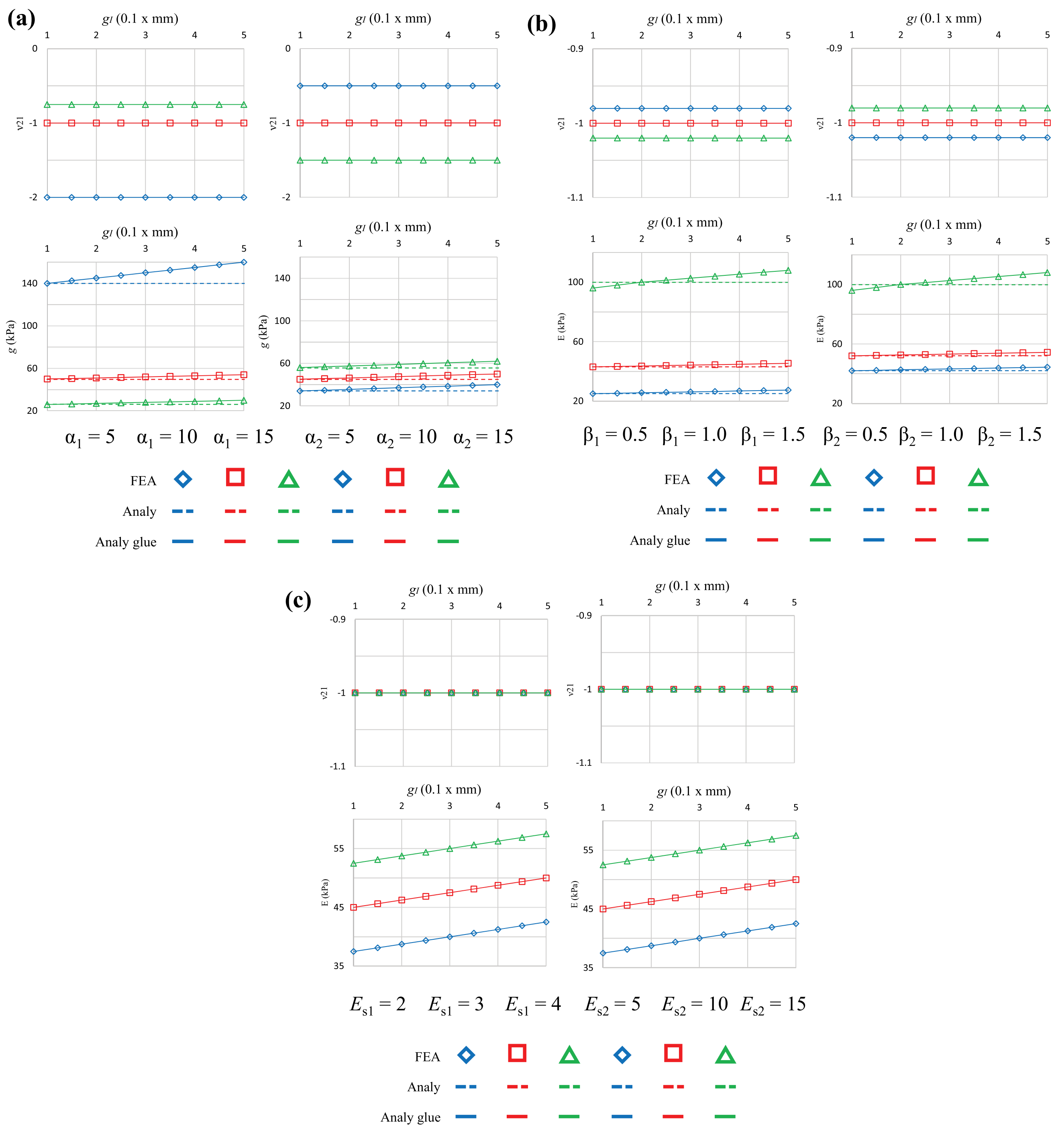

Fig. 7. Plots showing the effect of gluing material on the Poisson's ratio and Young's Moduli. The parameters used for each plot are given in Table 1. In (a), the effect of changing the parameters on the Poisson's ratios are shown. In (b), the effect on the Young's modulus of the system are shown. E_{si} are in GPa.

used, then it is more likely for the flexure mechanism to operate. In cases where the two modes of deformation act concurrently, then the observed Young's modulus will be equal to:

$$\frac{1}{E_{OBS}} = \frac{1}{E_{Flex}} + \frac{1}{E_{Hing}} \tag{15}$$

This equation, which is valid for systems where the hinging and flexure mechanisms act in parallel, clearly shows that the observed Young's modulus is expected to be less than that in systems where an individual mechanism operates, as illustrated in Figure 9.

Also, for systems in which more than one mechanism operates, the Poisson's ratio would be expected to be equal to:

$$\frac{\nu^{obs}}{E^{obs}} = \frac{\nu^{Hing}}{E^{Hing}} + \frac{\nu^{Flex}}{E^{Flex}} \tag{16}$$

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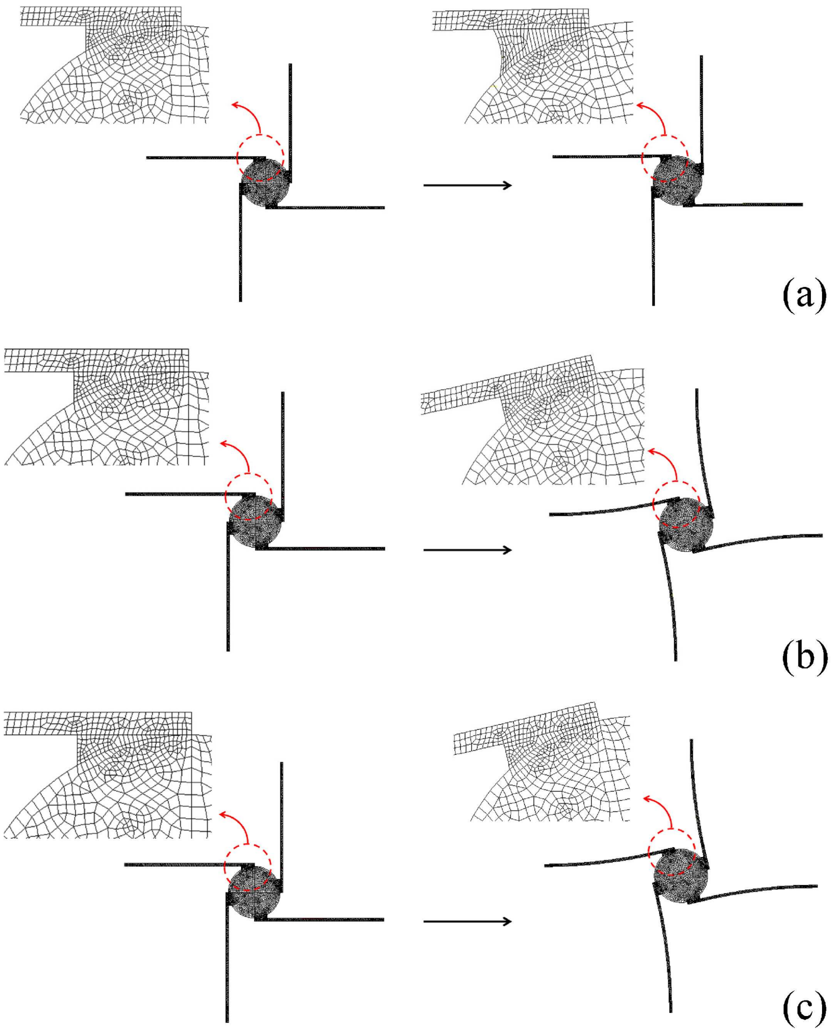

Fig. 8. A figure depicting the mode of deformation of the anti-tetrachiral system where the rigidity of the glue g_I was (a) 4×10^3 Pa, (b) 4×10^9 Pa, and (c) 4×10^{12} Pa. These systems were subjected to a strain of 10%. Note that in (a) where the glue is much softer than the rest of the structure, the system is deforming primarily though a "hinging mode of deformation" with the glue acting as the hinge. In the systems with stiff glue, (b) and (c), there are minimal deformations in the glue portion, so most of the deformations are in the form of flexure of the ligaments.

where for the particular cases considered here, since ν^{Hing} is equal to ν^{Flex}, $\nu^{\text{obs}} = \nu^{\text{Hing}} = \nu^{\text{Flex}} = -1$ (Figure 9).

This highlights the fact that the properties of the glue may have very significant effects on the properties exhibited by the structure. If the systems designed here were to be manufactured and commercialized, the aspect of how the ligaments are attached to the nodes should be considered in great detail. It should also be emphasized that this has been only a

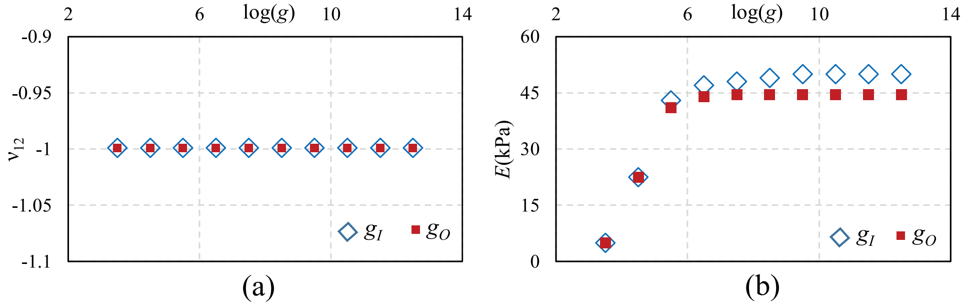

Fig. 9. Plots showing (a) the Poisson's ratio and (b) the Young's moduli for different glue rigidities. These plots confirm that the Young's moduli decrease as the glue becomes softer, as expected, since in such cases, the hinging mechanisms will start to become more dominant. Note that the Poisson's ratios remain −1 despite the change in deformation mechanism (Both hinging mechanism and flexing mechanism predict a Poisson's ratio of −1 for antichiral systems having the same properties and dimensions for the horizontal and vertical ligaments.)

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superficial treatment of the effect of the properties of the glue. It is well known, for example, that when two objects are glued together, the point of gluing is usually a "weak point" in the system. Furthermore, due to the high thickness and dimensions of the base glues, shear deformations of the glue might have important effects on the individual elements of the structure. Such effects are not being considered in this preliminary study, since it is being assumed that no failure of the connections occur.

4. Conclusion

It has been shown that the amount of gluing material used to attach the ligaments to the node will not affect the Poisson's ratio, but may have a large influence on the observed stiffness of the structure (Young's modulus), depending on the position and properties of the glue. Glues with a stiffness comparable to that of the nodes which are applied on the inner section of the ligaments were found to shorten their effective length, since the part of the ligament which is glued cannot bend. This difference was found to be highly significant only when the ligament length to node radius ratio approaches one, and becomes insignificant as the latter ratio becomes very large. This is because for small ligament length to node radius ratios, the part of the ligament that is glued (not allowed to move) is significantly large, whilst for large ligament length to node radius ratios, the part of the ligament which is glued is insignificant. No such effect was found for the glue applied to the outer side of the ligaments.

Furthermore, it was shown that the stiffness of the glue will have a large effect on the mode of deformation of the chiral system. A hinging mechanism is obtained when a "soft glue" is used, while a flexing mechanism is obtained when a "hard glue" is used. This change in mechanism was found to effect the stiffness of the structure but not its Poisson's ratios.

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