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On the mechanical properties of centro-symmetric honeycombs with T-shaped joints

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Centro-symmetric cellular honeycombs, in particular, their re-entrant version have been the subject of much research in recent years in view of their highly desirable macroscopic properties, including negative Poisson's ratio (auxetic) behavior. This work re-examines the mechanical properties of such honeycombs at the transition between the re-entrant and non re-entrant form, i.e. when the joints form a T-shaped junction, including the more general cases when the vertical ligaments have different thickness and material properties from the horizontal ligaments. It was shown through FE simulations that, contrary to current understanding, such honeycombs may deform in a manner which results in bending of the horizontal ligaments upon loading in the horizontal direction, with the result that such honeycombs may exhibit non-zero Poisson's ratio values. This effect was explained in terms of uneven stretching of the horizontal ribs upon uniaxial horizontal loading, a property which may also be employed to design honeycombs with negative Poisson's ratios.

1 Introduction Cellular materials have been the subject of much research in recent years in view of their excellent weight to stiffness characteristics which makes them ideal for use in applications where light weight materials are required. Moreover, it has been found that certain cellular solids may also exhibit auxetic behavior [1–20], the counter-intuitive property of becoming fatter when stretched, a property, which is known to impart a number of beneficial qualities [21–27].

Amongst the known auxetic systems [1–20, 28–50], hexagonal honeycombs were one of the earlier systems to be studied for such behavior, with a geometry that allows for the tuning of the Poisson's ratios to a whole wide spectrum of values ranging from positive to negative. Various studies have been conducted with the aim of studying the mechanical properties of hexagonal honeycomb-like structures having arrow shaped (re-entrant) or Y-shaped (classical non re-entrant) joints, see Fig. 1a and b [1–5, 51]. For example, in their pioneering work, Gibson and Ashby [1, 2] have suggested that the in-plane moduli and Poisson's ratios of such honeycombs may be predicted through simple models. The model proposed by Gibson and Ashby's in their 1982 work is based on the assumption that the honeycombs deform solely through flexure of their ribs and, despite its relative simplicity, is known to give an excellent first approximation of the mechanical properties of real honeycombs [1]. More recently, Gibson and Ashby [2] have also proposed a more extended model that also accounts for axial and shear deformation of the cell walls, which model may be regarded as being more representative of the true behavior of cellular honeycombs.

Another highly realistic model was also suggested by Evans et al. [3, 4] where, in addition to the flexure mode of deformation, the honeycombs were also permitted to deform through stretching of the ligaments and a hinging-type mechanism which involves changes in angle between the ligaments [3, 4]. This model was more recently extended by Grima et al. [5] who argued that, in the vicinity of the joints, there are only negligible deformation with the result that the real length of the ribs which is actually deforming is shorter than the length of the rib one normally measures and therefore, one should consider the deformation of an effective length rather than the actual length. Such multi-mode models [2–5] are particularly useful in geometries where the contribution of these deformation mechanisms (i.e. stretching and hinging) is comparable to that of flexure and therefore cannot be neglected. For example, at small angles, the stretching of the ligaments is expected to be non-negligible, in which case, these multi-mode models would give significantly better predictions.

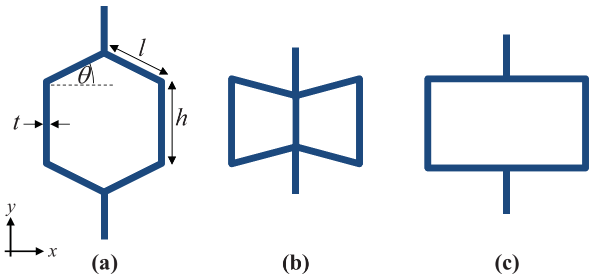

Figure 1 Different possible configurations for centrosymmetric hexagonal honeycombs with (a) Y-shaped joints, (b) arrow shaped joints and (c) T-shaped joints.

The suitability, or otherwise, of any model to predict the true behavior of a real system is invariably dependent on the assumptions the model is built upon, and obviously, this is also the case for the analytical models of these hexagonal honeycombs. It is a well-known fact that any model may not work equally well under all circumstances, as is unfortunately the case for the models of the hexagonal honeycombs. In fact, it may be easily shown that although the published analytical models, particularly the more recent ones, give an excellent prediction of the mechanical properties of most honeycomb-like systems, the models may have limitations to account for the deformations and properties in honeycombs where the joint is T-shaped, (i.e. the transition point between re-entrant and the classical non re-entrant honeycombs where $\theta = 0$). In fact, preliminary static simulations using plane elements via the finite element analysis modeling software Ansys V13, suggest that for a honeycomb with geometric parameters $\theta = 0°$, $h = 20$, $l = 10$, $b = 1$, and $t \in \{0.1, 0.2, ..., 1\}$, and built from a framework with mechanical properties of, $E_s = 10$ GPa, $\nu_s = 0.3$, the Poisson's ratio ν_{xy} is simulated to be approximately equal to 0.1 (refer to Fig. 2), a value which is significantly different from the zero value predicted by the analytical models derived by Masters and Evans [4], Gibson and Ashby [2], or Grima et al. [5], especially when

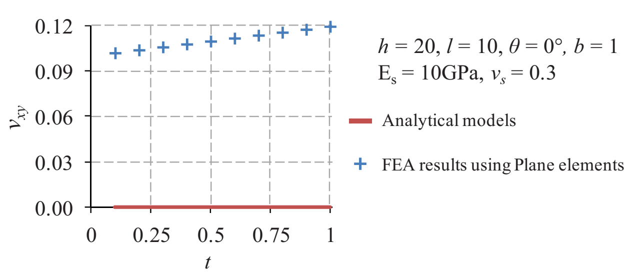

Figure 2 ν_{xy} as predicted by FEA simulations using plane elements and the analytical models of Masters and Evans [4], Gibson and Ashby [2], and Grima et al. [5], for different values of t, at $\theta = 0°$. The analytical models predict a value of zero for any value of t, while the simulations predict a non-zero value, which appears to be dependent on the thickness of the ligaments.

one bears in mind that the Poisson's ratio of most isotropic solids is *ca*. 0.3.

In view of such discrepancies, which as yet have been largely overlooked, this work presents a detailed study on hexagonal honeycombs for the particular cases where the honeycomb has T-shaped joints (i.e. at the point where θ is zero). In particular, this study looks in more detail at the in-plane mechanical properties of these systems, especially the Poisson's ratio ν_{xy}, when finite strains are applied along x. More specifically, finite element simulations shall be carried out on these systems so as to fully understand the properties they manifest. Furthermore, possible structural adjustments of these frameworks so as to make them amenable to a deformation mechanism, which has the potential to exhibit a negative Poisson's ratio shall also be proposed.

2 Simulations Honeycombs with T-shaped junctions having a unit cell as illustrated in Fig. 3a were simulated using the finite element modeling software ANSYS, V13 running on a Fujitsu Celsius R570 with 32Gb RAM and 2 Intel Xeon X5650@2.67GHz Processors. The unit cell size was chosen since preliminary simulations have shown that such size is most appropriate for simulating the properties of such systems in a realistic and reproducible manner without excessive computational burden. In analogy to previous work by Grima et al. [5], the simulations were performed using a planar element (2D 8-Node PLANE223), which assumes a quadrilateral shape which however may be changed to a triangular shape if required, and has mid-nodes along each side resulting in a total of eight nodes per element. This element allows for displacement along x and y directions caused by mechanical and/or thermal loading. The honeycombs were made from a single material comparable to that of steel ($E_s = 215$ GPa $\nu_s = 0.3$), thus enabling representation of a realistic system.

To allow for a better understanding of how the various geometric parameters (refer to Fig. 3) affect the properties of these honeycombs, six sets of simulations were performed, where in each set, all but one of the geometric parameters were kept constant, as follows:

(a) $l \in \{4, 6, ..., 22\}$, $h_{\mathrm{eff}} = 10$, $t_l = 0.2$, $t_h = 1$,
(b) $l = 10$, $h_{\mathrm{eff}} = 10$, $t_l \in \{0.05, 0.10, ..., 1\}$, $t_h = 1$,
(c) $l = 10$, $h_{\mathrm{eff}} = 10$, $t_l = 0.2$, $t_h \in \{0.5, 1.0, ..., 9.5\}$,
(d) $l = 10$, $h_{\mathrm{eff}} \in \{2, 4, ..., 20\}$, $t_l = 0.2$, $t_h = 1$,

where all measurements are in arbitrary units, since the properties being measured do not depend on the scale used.

Moreover, a further case was considered where the vertical and horizontal ligaments were given a different Young's modulus, in this case the Young's modulus of the horizontal ligament was kept constant at 300 GPa and the modulus of the vertical ligament was changed by first setting it at a value of 0.0001 GPa and then varied from 50 to 600 GPa with regular increments of 50 GPa, for a system having geometric parameters, $l = 10$, $h_{\mathrm{eff}} = 10$, $t_l = 0.2$, $t_h = 1$. Here, it should be emphasized that this system is not

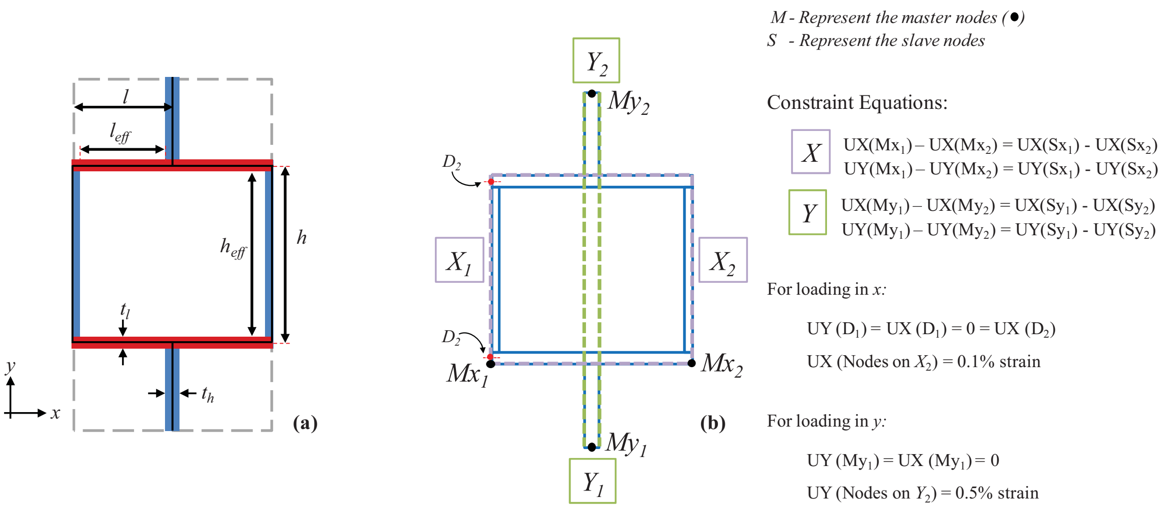

Figure 3 (a) The unit cell chosen for simulating these systems. Note that as $t \to 0$; $l_{\mathrm{eff}} \to l$, and $h_{\mathrm{eff}} \to h$, and the honeycomb reduces to the one shown by the solid black lines. The red rectangles indicate the horizontal ligaments, while the blue rectangles indicate the vertical ligaments. (b) The boundary conditions applied to the system to ensure tessellatability of the unit cell.

dissimilar to those studied by Grima et al. using plane elements via the FEA modeling so as to validate their analytical model, with the only difference being that this particular case, where $\theta = 0°$, was not specifically studied, or was not reported. Also, the systems modeled by Grima et al. [5] only considered ligaments with the same thickness t for the horizontal and the vertical ligaments and these were always constructed from materials having the same material properties.

Once these honeycomb systems were constructed within the ANSYS environment, each component of the system was meshed separately with the vertical ligaments having a maximum element size of $t_h/16$, and the horizontal ligament having a maximum element size of $t_l/8$. In this way, vertical ligaments were divided in at least 16 divisions along their thickness, while the horizontal ligaments, being in most cases thinner, were divided in at least eight divisions, while the number of elements along the length of the ligaments depended on the magnitude of the respective geometrical parameter. Thus, the number of elements and nodes used to represent each system was dependent on the size of the system. Moreover, the meshing was performed such that a smaller element size could be automatically applied if necessary. This mesh sizing was found, through a convergence test, to be appropriate for adequately representing such systems, (i.e. where the difference between consecutive finer meshes fell below 1%). It should be emphasized here that identifying the correct mesh is very important as the outcome of the results is highly dependent on how well the mesh represents the system, as recently shown in work by Ref. [52] Also, the lines opposite each other at the boundaries of the unit cell taken were meshed with equal number of element divisions per line, which number depended on the element size chosen. This allows for the ease of implementation of periodic boundary conditions which were applied so as to simulate an infinite and tessellatable system by constraining the resulting nodes at the opposite edge boundaries in such a way to ensure that opposite sides of the unit cell behaved in an identical manner so as to retain tessellatability. In particular, referring to Fig. 3b, any other node apart from the master node, on lines X_1, X_2, Y_1, Y_2 was set as a slave node where each pair of slave nodes, were chosen such that, Sx_1 lies on line X_1 and Sx_2 lies on line X_2 while each pair had the same value of y. Similarly, Sy_1 lies on line Y_1, Sy_2 lies on line Y_2 while each pair had the same value of x. Each pair was coupled together with the master node as stated by the equations for coupling in Fig. 3b.

Furthermore, in order to prevent the whole system from translating or rotating, rather than deforming, (i.e. rigid body motion) a number of nodes were prevented from moving along the x and y directions. These constraints were applied as shown in Fig. 3b. Following meshing of the structure and applications of periodic boundary conditions via constraint equations, simulations were then performed where the structure was subjected to a fixed uniaxial strain (0.1% in the case for loading in x direction and 0.5% in the case of loading in the y direction) and solved linearly so as to assess how the structure would deform under uniaxial loading conditions. Strains in the orthogonal directions were then estimated from the deformations of the system, which were then used to calculate the Poisson's ratios.

3 Results and discussion An analysis of the deformed and undeformed structures from the simulations performed suggests that, upon uniaxial loading in tension in the x direction, the initially straight horizontal ligaments deform in a manner to make them resemble the geometry of re-entrant honeycombs, (henceforth referred to as the T-joint effect) as one may clearly observe from Fig. 4, which shows images of a typical deformed and undeformed structure. This deformation resulted in a positive Poisson's ratio having non-insignificant values, which cannot be approximated to zero, as clearly illustrated in the plots shown in Fig. 5. This result, which had so far remained unnoticed, was *prima facie* unexpected since, conventional reasoning would make one expect that loading a ligament-like unit along its length should only result in stretching of the ligament.[a] In fact, this was the main reasoning behind the model presented by Masters and Evans [3, 4] who suggest that honeycombs having T-shaped joints were expected to deform primarily through stretching of the horizontal ligaments thus resulting in a Poisson's ratio $\nu_{xy} = 0$, which is clearly not the case.

For loading in the y direction, no major deviations for the Poisson's ratio ν_{yx} were observed from that predicted by Masters and Evans [4] and Grima et al. [5], since the simulations confirmed the existing analytical models predictions that ν_{yx} should be equal to zero. (The simulations predict a Poisson's ratio to be in the order of *ca.* 1×10^{-3} and indeed vary with varying geometric parameters, however the very small values may be approximated to zero). Indeed if one looks at the deformed structures obtained for loading in y, the horizontal ligament undergoes considerable deformation, however the geometry is such that the Poisson's ratio ν_{yx} still remains approximately zero as expected.

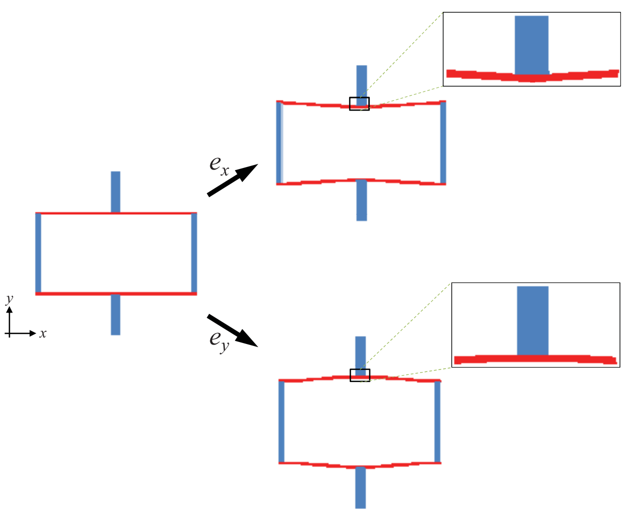

Figure 4 The deformed and undeformed structure for a typical system for loading by a 5% tensile strain in x and y, suggesting that for loading in x, non-uniform deformations occur in the region of overlap of the vertical and horizontal ligaments, indicating that deformations other than simple stretching occur in this region, resulting in the observed curvature of the horizontal ligament at this point of overlap.

In order to better understand how these systems are undergoing such unexpected deformation for loading in the horizontal x-direction, one needs to look in more detail at the images of the system under a uniaxial tensile load as depicted in Fig. 4. From this figure, it is immediately evident that deformations other than simple uniform stretching occur in the region of the joints, as opposed to the other regions of the horizontal ligaments. This suggests that, although simple uniform stretching is most probably the major mode of deformation in the main part of the ligament, this is not the case in the region where the vertical and horizontal ligaments intersect. In such a region, a non-uniform stretching of the ligaments appears to be the major mode of deformation which gives rise to a hinging-type of deformation, effectively resulting in an in-plane rotation of the segment of the horizontal ligament between the two vertical ligaments by an angle θ. This mode of deformation may be mainly attributed to uneven stretching deformations that take place because of the added thickness on one surface of the horizontal ligament (due to the presence of the vertical ligament), which makes the common surface much less amenable to deformations than its opposite surface. This uneven thickness effectively prohibits stretching on the more rigid surface (i.e. the surface where the vertical ligament is attached) and only permits stretching at the other surface resulting in the observed uneven stretching. This uneven mode of deformation at this particular region results in curvature of the "free" surface since it is able to undergo more deformation. Here, it should be noted that such uneven stretching of these different surfaces of the horizontal ligament is not dissimilar to what is observed in bimaterial strips where a curvature is observed upon heating due to an uneven extent of thermal expansion of the different materials resulting in the "free" surface expanding more than the "constrained" surface. A proof of this principle was obtained from the highly simplified physical model constructed from silicone material shown in Fig. 6 in its stretched and unstretched states, where the deformations described above are evidently visible.

To verify the hypothesis proposed here, i.e. that the presence of the added constraints on one surface of part of a ligament results in its curvature upon loading along its length, additional simulations were performed on simplified systems as shown in Fig. 7. These systems are meant to represent a rectangular ligament-like system of dimensions $l \times t_l = 20 \times 0.5$ made from an isotropic material having Poisson's ratio 0.3 and Young's modulus $E_s = 215$ GPa which has been stretched via the application of forces F_x (where $F_x = \pm 0.1 E_s t_l$) applied to both ends of the ligament-

[a] The model by Masters and Evans [4] was based on the assumption that for loading in the x-direction, since the horizontal ligaments lie in the direction of loading and the vertical ligaments lie in a direction which is perpendicular to it, then it is only the horizontal ligaments which experience the load, i.e. the vertical ligaments may be ignored. With such assumptions, one may be led to assume that the system could be effectively approximated by a series of horizontal ligaments, with the vertical ligaments playing no role in the deformation.

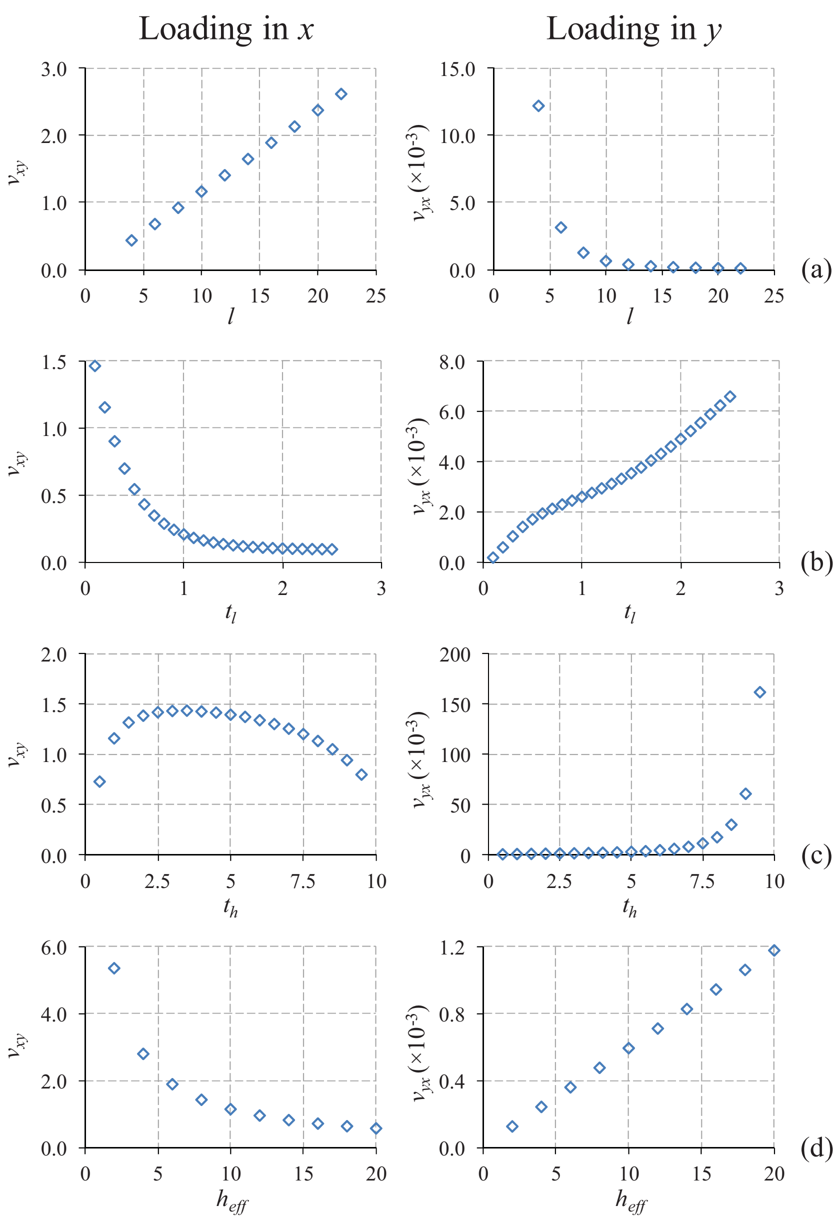

Figure 5 The Poisson's ratio obtained from FEA simulations for the honeycomb systems at $\theta = 0°$, for the system described in Fig. 3 having typical values of $l = 10$, $h_{\mathrm{eff}} = 10$, $t_{\mathrm{l}} = 0.2$, $t_{\mathrm{h}} = 1$, apart for the varying parameter, where the changing parameter had values as described in the methodology for the respective graphs. Note the difference in scale for ν_{xy} and ν_{yx}.

like unit along its length. This unit was additionally being constrained in such a way that:

Case I: A portion of the top surface is clamped in the x and y direction to prohibit it from deforming;

Case II: A portion of the bottom surface is clamped in the x and y direction to prohibit it from deforming;

Case III: A portion of the top surface is clamped in the x and y direction to prohibit it from deforming whilst an equal portion on the opposite bottom surface is clamped in the x direction to prohibit it from deforming along the length whilst still permitting deformations along the y-direction.

As clearly illustrated in Fig. 7, which shows the deformed systems I–III; it is evident that, pulling in the direction of the length of the ligaments, when either the top or bottom surface portions, but not both together, are prohibited from moving (i.e. in case I and II but not in III), will result in, not only an increase in the length of the system, but more importantly, a curvature in the "free" surface due to the larger deformation it exhibits, in an analogy to what was hypothesized above. In Case III, no such curving occurs and the ligament was simply observed to elongate in the direction of pulling whist becoming thinner in the orthogonal direction, as expected from an unconstrained or symmetrically constrained ligament-like unit made from a material having a positive Poisson's ratio (i.e. a ligament undergoing a uniform stretching mode of deformation).

Moreover, simulations in which the Young's moduli of the vertical and horizontal ligaments were altered also appear to suggest that this mechanism is in fact taking place, since on increasing the Young's modulus of the vertical ligament,

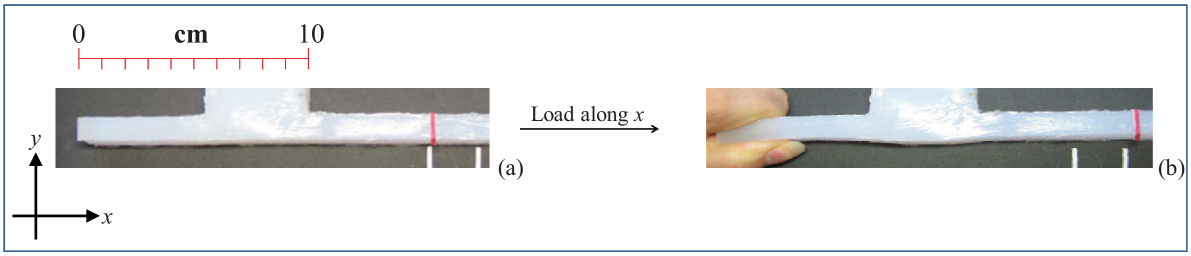

Figure 6 The undeformed (a) and deformed (b) photos of a simplified physical model built from silicone material. On stretching along the length of the horizontal ligament, the response described above, where the shorter rectangular block extending from the top surface of the longer block is prohibiting this surface from deforming, is clearly evident.

the Poisson's ratio ν_{xy} increases in magnitude, as shown in Fig. 8. This is concordant with the hypothesis made here that the added stiffness provided by the vertical ligaments is causing the behavior being noted here.

At this stage, having established the cause of the unexpected behavior noted here, it would be useful to attempt to obtain a better understanding of the dependency of the macroscopic properties on the geometric parameters of the honeycombs. It should first of all be noted that all of the geometric parameters affect, in some way or another, the magnitude of the Poisson's ratio ν_{xy}. For example, the longer the length of the horizontal ligament, for a given rotation, the greater the displacement in the y direction is, hence resulting in an increase in Poisson's ratio ν_{xy} (refer to Fig. 5a). This increase in the Poisson's ratio may be explained by the fact that the y-displacement is given by $l_{\text{eff}}\sin(\theta)$ where θ is the angle that the horizontal ligaments effectively rotates, and, l_{eff} is the effective length of the horizontal ligament, i.e. the length of the free-portion of the ligament (that lying between the vertical ligaments), in this case assumed to be $l_{\text{eff}} = l - t_{\text{h}}$.

The thickness t_{l} of the horizontal ligament parallel to the x-axis was found to play an important role on the degree and extent of deformation that the system exhibits. In fact, as shown in Fig. 5b, for an increase in the horizontal ligament thickness t_{l}, the Poisson's ratio, ν_{xy} was predicted by the simulations to decrease and approaches zero, when the systems were subjected to a tensile load along x, in other words approaching the behavior assumed in earlier work by Masters and Evans [4] and Grima et al. [5]. This result is to be expected and may be explained by the fact that an increase in the thickness of this ligament decreases the curvature that these ligaments can undergo, since a larger force would be required to bend this thicker ligament resulting in less curvature being observed for the same forces being applied, and thus this observed deformation is reduced to the extent that the Poisson's ratio approaches zero.

The relationship between the Poisson's ratio ν_{xy} and the thickness t_{h} of the vertical ligaments is more complex. In fact, having established that the region where the vertical and horizontal ligaments intersect exhibits the major non-uniform deformations, it is understandable that, as the overlap in this region was increased by increasing the thickness of the vertical ligament t_{h}, one would observe an increase in the rotation angle θ, as observed from Fig. 9a, since there would be a larger portion of the surface of the horizontal ligament which is constrained not to deform. However, this increase in curvature upon increasing t_{h}

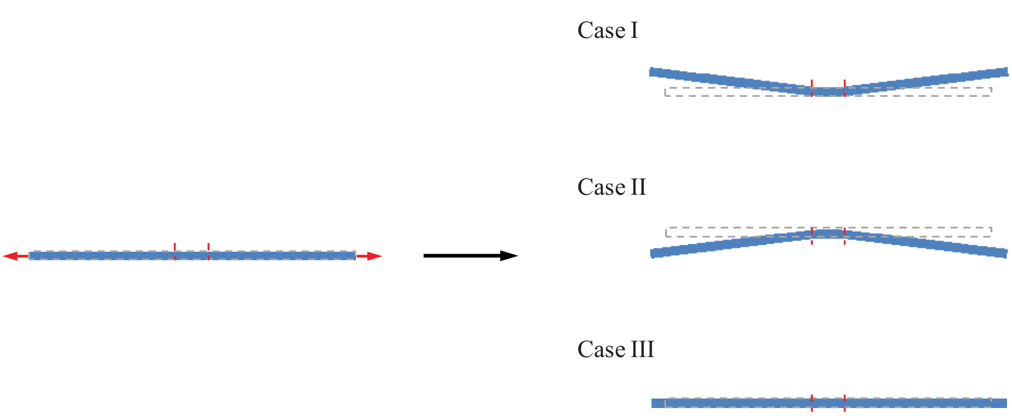

Figure 7 The deformed and undeformed systems for cases I–III, the red dotted line highlight the region over which constraints where applied, on either the top/bottom nodes or both as discussed in text, where it is evident that uneven constraints are affecting the mode of deformation of the ligament.

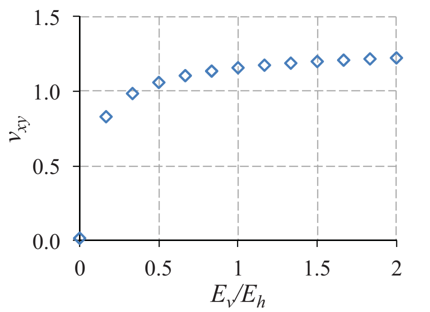

Figure 8 Figure showing the relationship between the Poisson's ratio, ν_{xy}, against the ratio of the Young's moduli, E_v/E_h, for a system having typical values of $l = 10$, $h_{eff} = 10$, $t_l = 0.2$, $t_h = 1$, where E_v is the Young's modulus of the vertical ligament and E_h is the Young's modulus of the horizontal ligament.

seems to reach a threshold value, as evident in Fig. 9b, after which an increase in t_h would not result in such a large increase in θ as was observed before the threshold value, as evident in Fig. 9c.

Such a discontinuity in the trend of increase in θ for an increase in t_h was somewhat unexpected but may be easily explained by the fact that, since h_{eff} is relatively large (compared to the thickness of the horizontal ligament t_l, as evident from Fig. 9), the vertical ligament undergoes only slight deformations, most of which are restricted mainly to a slight curving near the edges of the region of overlap. This implies that an initial increase in the thickness of the vertical ligament would better accommodate such curving deformations and thus, since there is more curvature, the angle θ increases. However, once this curving deformation reaches its full potential, further increase in the thickness beyond the threshold value would only affect slightly these deformations and thus would result in a smaller change in θ as evident from Fig. 9c. Moreover, bearing in mind that the y-displacement which gives rise to the non-zero Poisson's ratio ν_{xy} is given by $l_{eff}\sin(\theta) = (l - t_h)\sin(\theta)$, it becomes evident that the Poisson's ratio ν_{xy} may only increase (due to an increase in θ, which overcomes the increase in t_h) up to a certain point, after which it starts again to decrease as a result of the increase in t_h which reduces the value of l_{eff}. Further evidence to this observation can be obtained using a simple model where portions of the vertical ligaments are removed such that this ligament assumes an I-beam shape as shown in Fig. 10 with the aim of maintaining the same length of contact while giving additional flexibility to the region of overlap. It is evident that such an adjustment results in an increase in the Poisson's ratio observed for an increase in the region of overlap (t_h), even beyond the threshold value, as opposed for the honeycomb system, indicating that the extra material of the vertical ligament is indeed prohibiting the deformation to be less pronounced beyond this value.

It is also interesting to observe that, as shown from Fig. 5d, the Poisson's ratio when loading in the x direction is dependent on the length h of the vertical ligaments, a property which is easily understood since, $\nu_{xy} = -\Delta y/y\varepsilon_x$ where $\Delta y = l_{eff}\sin(\theta) = (l - t_h)\sin(\theta)$ is independent of h as opposed to $y = 2h$. Thus, it is to be expected that ν_{xy} increases on decrease in h, since these two properties are inversely proportional to each other. From a structural point of view, this may be explained by the fact that the density of the junctions (where each junction contributes to the overall deformation) decreases for a system having higher values of h, resulting in a decrease in the Poisson's ratio due to this decrease in the number of junctions rather than due to an intrinsic interplay in the deformation mechanism.

At this point, it should be highlighted that the results and effects reported here could not have been observed if one was to perform FEA studies using the simpler and more traditional methods of simulations based on beams which give a simulated value of 0 for ν_{xy}. This inability to capture the effects being reported here for the first time further emphasize the need to improve the beam formulation in FEA methods so as to permit a more accurate representation of the behavior at junctions between beams. In the absence of such improvements, the results obtained from beam FE methods should be treated with caution.

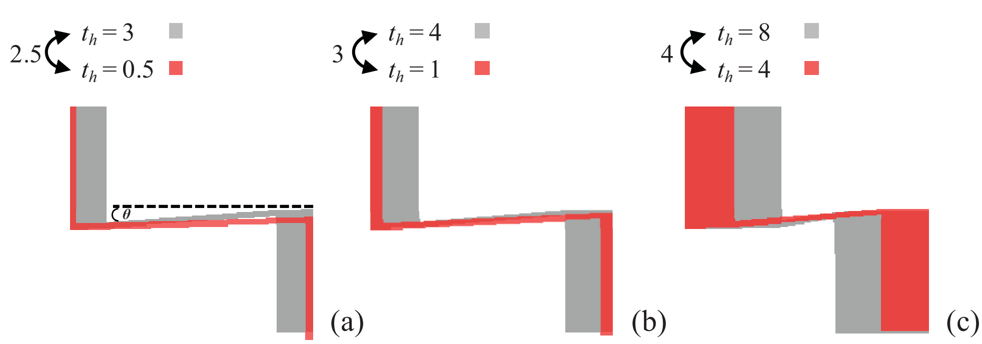

Figure 9 Superimposed images of a quarter of the unit cell for systems with different values of t_h after a 0.1% strain was applied on the system, with a displacement scaling of ×40 of the original, showing that (a) at first for an increase in t_h there is an increase in θ, then beyond a point (b), the increase becomes less pronounced and (c) the increase in θ is not large enough to result in a system with a larger Poisson's ratio.

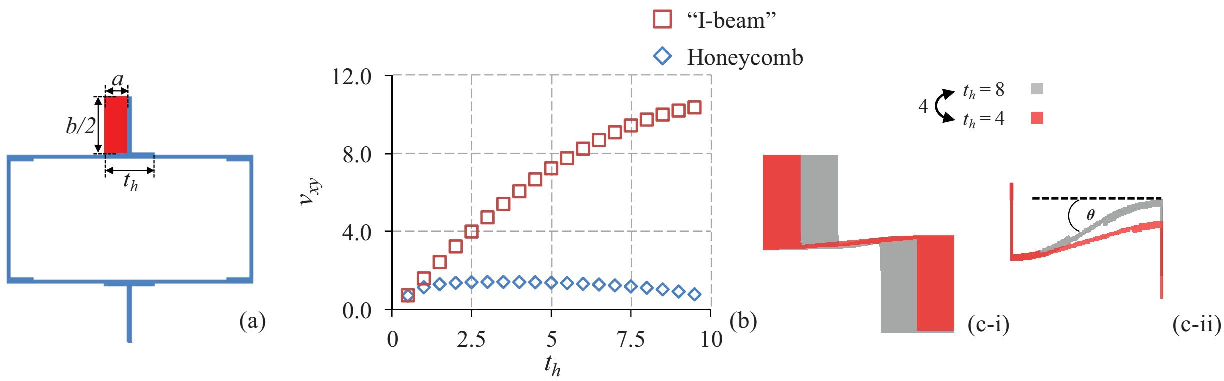

Figure 10 (a) The honeycomb built from I-beams, where the red rectangle ($a \times b$) was removed from the vertical ligament to maintain a consistent overlap as in the centro-symmetric hexagonal honeycomb system being studied but increase the flexibility of this region (for this particular case, $t_h = 0.5, 1.0, 1.5, \ldots, 9.5$, $a = 0.5(t_h - 0.2)$, $b = 9.8$, such that both the vertical and horizontal ligaments of the I-beam had a thickness of 0.2). (b) The graphs obtained for changes in t_h by both systems, where it is evident that for the system built from I-beams the Poisson's ratio continues to increase since, as opposed to the honeycomb system (c–i), the angle θ continues to increase considerably for any value of t_h studied (c–ii). Both systems have ×40 displacement scaling.

Before concluding, it is important to note that the results observed here, although very interesting and unusual, need to be investigated further. For example, this work is mainly based on simulations performed using linear finite element analysis. Whilst such simulations may be enough to provide a proof of concept, it is essential that this work is supplemented by a set of detailed experiments, or non-linear based analysis so as to investigate more thoroughly the work presented here.

Also, it is important to highlight that the main observation made here, i.e. that a ligament which is pulled along its length will curve if portions of its surface are constrained not to deform, provides us with a new way forward for designing honeycombs with tailor made, possibly anomalous, properties such as negative Poisson's ratio. In fact, considering the basic structure discussed here, (i.e. a honeycomb structure made up of T-shaped junctions) the deformations observed for pulling along x, resulting in a non-insignificant positive Poisson's ratio, is being manifested due to the deformations of the T-junctions which are becoming arrow-shaped junctions (↑). This, as explained, is due to a mechanism involving uneven stretching of the top and bottom surfaces of the horizontal ligament. In other words, this type of behavior would not have taken place if this uneven stretching was eliminated. This could be done, for example, by placing a rectangular element, which would rigidify the free surface of the horizontal ligaments opposite to the vertical ligaments as shown in Fig. 11, resulting in a system with the potential to exhibit a zero Poisson's ratio for loading in this direction, if this ligament impose similar

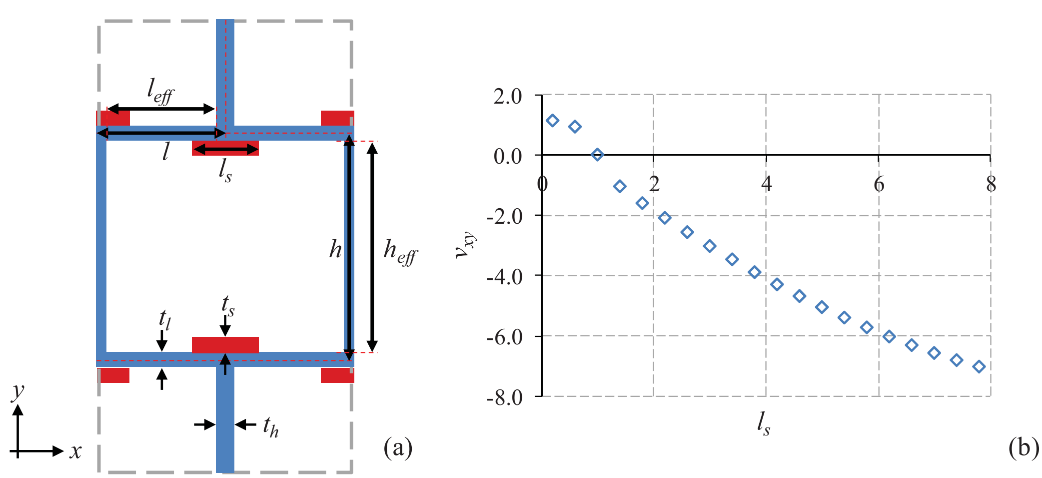

Figure 11 (a) The proposed system having a rectangular block on the surface of the horizontal ligament opposite to the vertical ligament, where t_s is the thickness and l_s is the length of the block, (b) graph of $_{xy}$ versus l_s, where the parameters already defined for the honeycomb system were taken as the typical values, i.e. $l = 10$, $h_{eff} = 10$, $t_l = 0.2$, $t_h = 1$, while t_s was set to 0.2 and l_s was changed as shown in the graph. The tunability of the Poisson's ratio this addition creates is clearly evident, where the Poisson's ratio may be changed to attain zero and negative values.

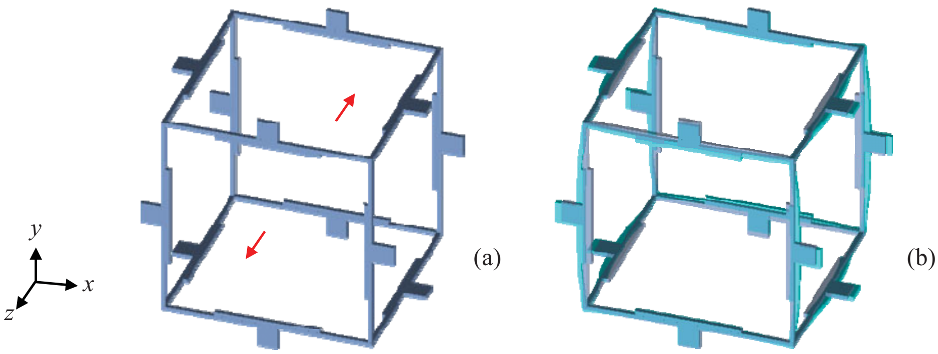

Figure 12 (a) One of the possible models one may build using this simple concept, where, upon uniaxial loading in z one may observe (b) an expansion in x resulting in a negative Poisson's ratio, ν_{zx}. One may also observe that the ligaments along z curve in both x and y which indicates that correct building of other systems may result in out-of-plane bending.

constraints on this surface as those imposed by the opposite vertical ligament. More importantly, if this rectangular element was made to impose more rigidity than the ligament itself, something which could occur through, for example, careful choice of the materials or geometric parameters used in constructing the system, then, the deformations would result in the joint becoming Y-shaped upon pulling rather than arrow-shaped resulting in a system which would exhibit negative, rather than positive Poisson's ratio for loading in the x direction as illustrated in Fig. 11.

Furthermore, although this work only considers two-dimensional honeycombs, it is more than likely that the effects discussed here would also have some effect on more complex 3D cellular solids, such as the ones shown in Fig. 12, should these systems be constructed in the appropriate manner. It could also be possible to use the concept presented here, to force 2D honeycomb-like systems to deform out-of-plane, provided that the geometry of the system, together with the geometric parameters, and the thickness in the third dimension are appropriately chosen. It is beyond the scope of this work to fully investigate these more complex systems, however given the many practical applications were materials and structures with tailor-made or negative Poisson's ratio can be employed, it is hoped that these preliminary results presented here will be enough to encourage other scientists and engineers to design new structures exhibiting the behavior discussed here.

4 Conclusions In this work, the special cases of the centro-symmetric honeycombs in the specific case where the joints are T-shaped and the vertical ligaments may have different thickness and material properties from the horizontal ligaments were re-modeled through FE simulations. It was shown that contrary to current expectations, such honeycombs may deform in a manner which results in curvature of the horizontal ligaments at the region of intersection of the vertical and horizontal ligaments, upon loading in the horizontal direction. As a result, such honeycombs have been predicted to exhibit Poisson's ratio values, which may differ from the "zero" value one expects should the systems deform solely through stretching of the horizontal ligaments. This effect was explained in terms of uneven stretching of the horizontal ligaments upon uniaxial horizontal loading. It was also shown that one may use the concept described here to design novel honeycomb based systems which may have anomalous properties with the result that one may even control, for example, the magnitude and sign of the Poisson's ratio. Moreover it was proposed that this concept may be used to develop other systems with tailor made and anomalous Poisson's ratios.

Given the practical importance of honeycombs in many everyday applications, as well as the usefulness of having a way to control the macroscopic properties of materials, it is hoped that this work will provide an impetus to scientists and experimentalists who may wish to further investigate the properties being predicted here, possibly through experimental work with the scope of understanding better the properties afforded by these systems.

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