

Auxetic behaviour from rotating rhombi

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Auxeticity (i.e. negative Poisson's ratios) is associated with particular geometrical features and deformation mechanisms of a system. Among the potential systems that may exhibit such behaviour are rotating rigid units. Here we discuss how rigid rhombi of the same shape and size can be connected in two different ways to give rise to two distinct systems: the Type α and Type β rotating rhombi. We derive the mechanical properties, for the Type β system and compare it to those of the Type α to highlight the differences between the two systems namely that Type α systems are highly anisotropic

and can exhibit both negative and positive Poisson's ratios which depend on the shape of the rhombi and the angle between them whereas Type β systems are isotropic with a constant in-plane Poisson's ratio of -1 . Furthermore we show that by constructing the rhombi from five truss-type elements, where the one forming the diagonal has a different thermal expansion than the other four identical trusses, the Poisson's ratios for the Type α systems can also be made temperature dependent, in contrast with Type β systems which remain unaffected by temperature changes.

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1 Introduction Among the fundamental mechanical properties used to quantify the response of a material when subjected to external stresses is the Poisson's ratio, defined as the negative of the ratio of the transverse and longitudinal strains. Basic thermodynamic theories show that for isotropic materials this ratio can take any values between -1 and 0.5 [1–3], with a larger range of values being possible for orthotropic and anisotropic materials. This suggests that although for most common materials the ratio has a positive value, and therefore reflects their tendency to contract when uniaxially stretched, materials with a negative Poisson's ratio (more commonly referred to as auxetic materials) can exist [1–32]. It is known that as a direct consequence to having negative Poisson's ratios, auxetic materials have some considerably improved properties when compared to conventional ones. In fact, studies show that they benefit from an enhanced resilience to indentation [4–6], a natural ability to form dome shaped surfaces [1, 4–6] and improved acoustical damping [7], making them useful for several engineering applications.

Although materials having a negative Poisson's ratio are much less frequent, they do occur in nature. Typical examples of such naturally occurring materials include mi-

nerals such as certain silicates (e.g. α -cristobalite [8–10]) and zeolites (e.g. natrolite [11–13]) and certain metals (e.g. some metals having a fcc [14, 15] and bcc arrangement [15] and certain alloys [16]).

Auxeticity is however not limited to the nanolevel but can also operate at the microlevel and also at the macro level resulting in auxetic structures. At any level, the auxeticity can be explained in terms of models describing the geometrical features of the system (features in the nano/microstructure in the case of materials) and the way these change when the material/structure is subjected to a uniaxial stress (the deformation mechanisms). This common characteristic allows the mapping of the larger scale macromodels down to molecular systems and vice versa, making it possible to elucidate and predict auxeticity at the nanolevel and also design new materials by constructing a molecular framework that mimics the same geometry and deformation mechanisms as the large scale auxetic structures. In fact, this approach has proved to be very useful in explaining auxeticity in α -cristobalite in terms of rotating tetrahedrons [17, 18] and/or rotating rectangles [19], in predicting auxeticity in natrolite through a rotating squares model [20] and also in designing hypothetical molecular

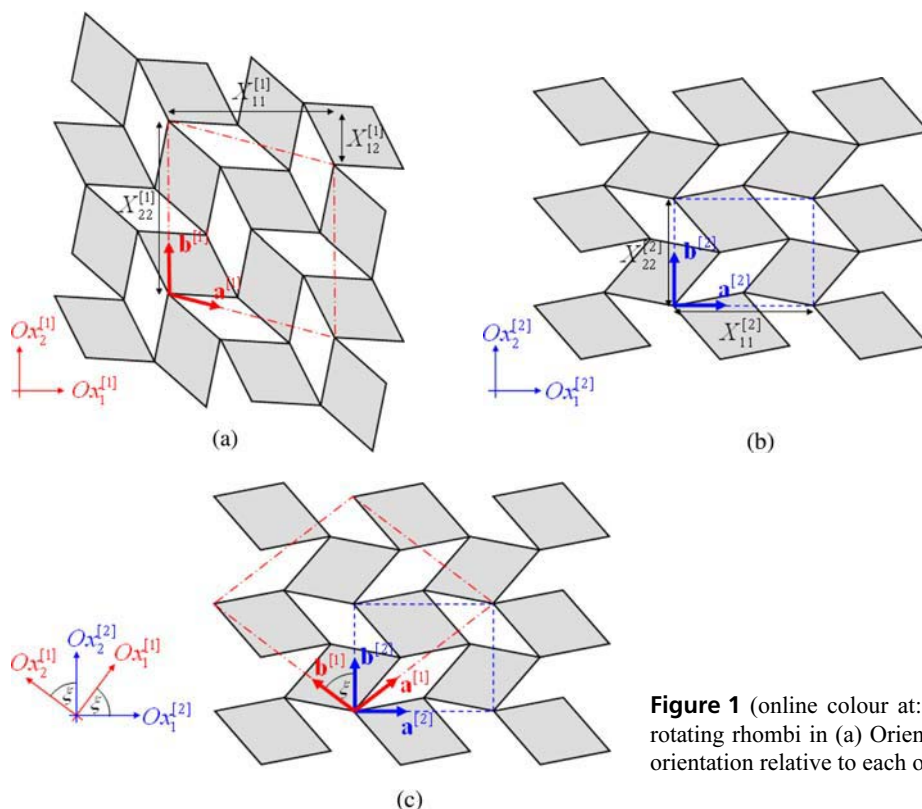


Figure 1 (online colour at: www.pss-b.com) Unit cell for Type α rotating rhombi in (a) Orientation 1, (b) Orientation 2, and (c) their orientation relative to each other.

networks based on various structures, in particular, re-entrant honeycombs [21] and rotating rigid units [22] through down scaling.

In this work, we review and extend the existing knowledge on the mechanical properties of two-dimensional rotating rigid quadrilaterals, in particular rotating rigid rhombi where we discuss how their mechanical properties are affected by the shape of the rhombi and the way they are connected. We also discuss how the geometry of these systems, and thus their properties can be made temperature dependent with the result that, in some cases, the extent of auxeticity can become a function of temperature.

2 Rotating rhombi: Analytical modelling

2.1 General considerations As discussed elsewhere [33], rhombi can be connected together at their vertices in two different ways to give rise to two non-equivalent networks which have been termed Type α and Type β . The topological feature which distinguishes these two types of networks is the magnitude of the internal angles of the rhombi at each hinge. In particular, as illustrated in Fig. 1, in ‘Type α rotating rhombi’ the obtuse angle of a rhombus always connects to an acute angle of an adjacent rhombus, while in ‘Type β rotating rhombi’ adjacent rhombi have their like angles connected to each other (i.e. an acute angle of one rhombus connects to an acute angle of another rhombus and an obtuse

angle of a rhombus always connects to another obtuse angle).

The geometry of these networks can be described by two differently oriented types of unit cells (henceforth referred to as ‘Orientation 1’ and ‘Orientation 2’, see Figs. 1 and 2) which are rotated by some angle ξ with respect to each other such that the diagonal of the unit cell in Orientation 2 coincides with the side of the unit cell in Orientation 1 as shown in Figs. 1c and 2c where Orientation 1 is defined as the one having the larger size enclosing four rhombi within it, while Orientation 2 is the smaller one containing only two rhombi¹.

2.2 The Type α rotating rhombi In our derivations and discussion we shall assume that in all cases the unit cell vector \mathbf{b} for any orientation used is always aligned parallel to the Ox_2 direction while \mathbf{a} is allowed to point in any direction in the plane of the network. Using these alignments we note that for the Type α rotating rhombi, the unit cell vectors in Orientation 1 are given by $\mathbf{a}^{(1)} = (X_{11}^{(1)}, X_{12}^{(1)})$ and $\mathbf{b}^{(1)} = (0, X_{22}^{(1)})$ while those in Orientation 2 are given by $\mathbf{a}^{(2)} = (X_{11}^{(2)}, 0)$ and $\mathbf{b}^{(2)} = (0, X_{22}^{(2)})$ where

¹ Note that the mechanical properties for each network can be derived and discussed using any other orientation. However these two orientations have been chosen because of the fact that they may be easily related to crystals having similar geometries, e.g. the (001) planes of THO and NAT may be directly mapped to the rotating squares system in ‘Orientation 1’ and ‘Orientation 2’ respectively.

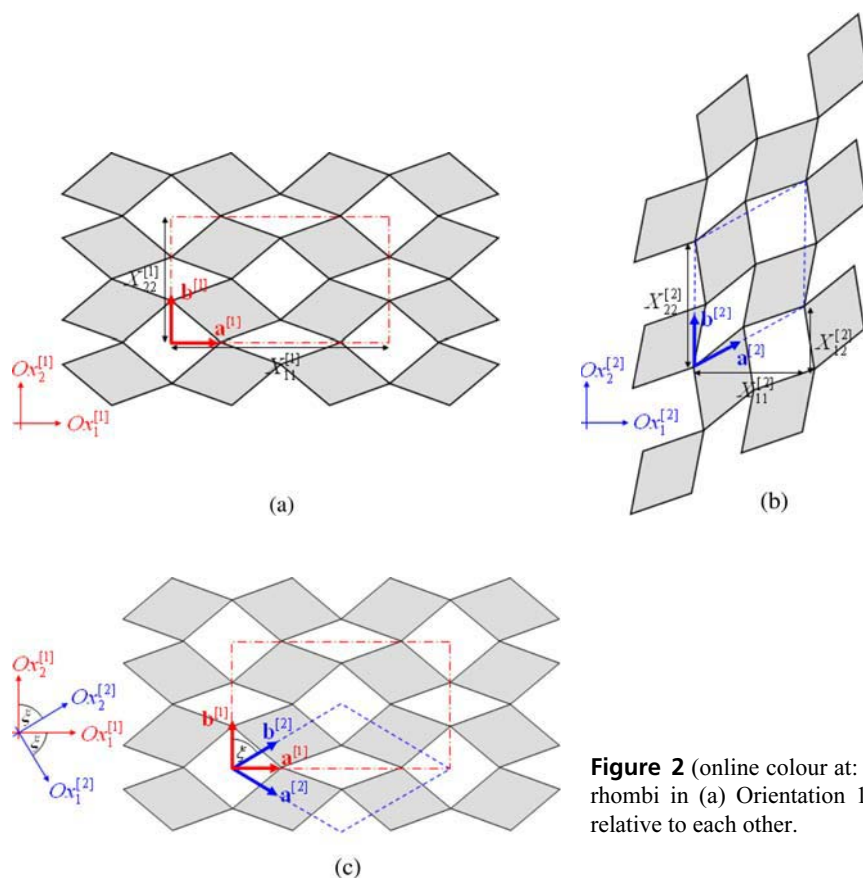


Figure 2 (online colour at: www.pss-b.com) Unit cell for Type β rotating rhombi in (a) Orientation 1, (b) Orientation 2, and (c) their orientation relative to each other.

the superscripts '[1]' and '[2]' indicate the orientation of the unit cell, and in particular for Orientation 2 (Figs. 1 and 3a):

$$X_{11}^{[2]} = 2a \sin\left(\frac{\theta + \phi}{2}\right), \quad (1)$$

$$X_{22}^{[2]} = 2a \cos\left(\frac{\theta - \phi}{2}\right), \quad (2)$$

where, as illustrated in Fig. 3a, a is the length of the side of the rhombus, ϕ is the internal angle of the rhombi and θ is the angle between the rhombi. The properties for the Type α rotating rhombi have already been derived elsewhere [33]

(as a special case of parallelograms) using 'Orientation 2' (the most appropriate orientation for deriving the mechanical properties of the Type α rotating rhombi), and it has been shown that the in-plane on-axis mechanical properties relative to Orientation 2 are given by:

$$\mathbf{S}^{[2]} = \begin{pmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{E_1^{[2]}} & -\frac{\nu_{21}^{[2]}}{E_2^{[2]}} & 0 \\ -\frac{\nu_{12}^{[2]}}{E_1^{[2]}} & \frac{1}{E_2^{[2]}} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

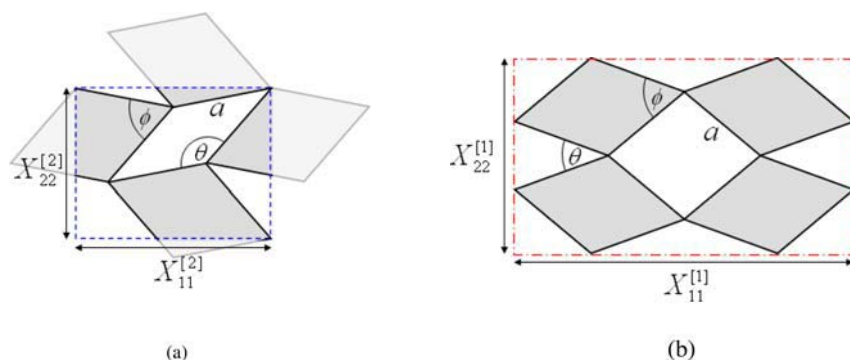


Figure 3 (online colour at: www.pss-b.com) Geometrical parameters describing the unit cell in (a) Orientation 2 for Type α rotating rhombi and in (b) Orientation 1 for Type β rotating rhombi.

where:

$$\nu_{12}^{[2]} = (\nu_{21}^{[2]})^{-1} = \tan\left(\frac{\theta - \phi}{2}\right) \tan\left(\frac{\theta + \phi}{2}\right), \quad (3)$$

$$E_1^{[2]} = \frac{4K_h \sin\left(\frac{\theta + \phi}{2}\right)}{a^2 z \cos\left(\frac{\theta - \phi}{2}\right) \cos^2\left(\frac{\theta + \phi}{2}\right)}, \quad (4)$$

$$E_2^{[2]} = \frac{4K_h \cos\left(\frac{\theta - \phi}{2}\right)}{a^2 z \sin^2\left(\frac{\theta - \phi}{2}\right) \sin\left(\frac{\theta + \phi}{2}\right)}, \quad (5)$$

where K_h is the stiffness constant associated with the hinges and z is the thickness in the third dimension so that when transformed, the in-plane off-axis Poisson's ratio

is found to be given by:

$$\nu_{12}^{[2]\zeta} = \frac{\cos\left(\frac{\theta - \phi}{2}\right) \cos\left(\frac{\theta + \phi}{2}\right) \sin^2(\zeta) - \sin\left(\frac{\theta - \phi}{2}\right) \sin\left(\frac{\theta + \phi}{2}\right) \cos^2(\zeta)}{\sin\left(\frac{\theta - \phi}{2}\right) \sin\left(\frac{\theta + \phi}{2}\right) \sin^2(\zeta) - \cos\left(\frac{\theta - \phi}{2}\right) \cos\left(\frac{\theta + \phi}{2}\right) \cos^2(\zeta)} \quad (6)$$

All this suggests that the Poisson's ratio of this system is dependent on:

- the shape of the rhombi (the angle ϕ),
- the angle between the rhombi (the angle θ),
- the direction of loading (the angle ζ , see Fig. 6 for a graphical illustration of the variation of the off-axis Poisson's ratio with the direction of loading)

Note that Eqs. (3)–(5) may be used to calculate the mechanical properties of the system in Orientation 1 by performing standard axis transformation of the compliance matrix and letting $\zeta = \xi = \cos^{-1}(X_{22}^{[2]}/X_{22}^{[1]})$.

The strains along the Ox_i directions can be expressed in terms of infinitesimally small changes $d\theta$ in the angle θ as:

$$d\varepsilon_i = \frac{dX_i}{X_i} = \frac{1}{X_i} \left(\frac{dX_i}{d\theta} \right) d\theta, \quad (9)$$

from which it follows that the on-axis Poisson's ratio are given by

$$\nu_{12}^{[1]} = (\nu_{21}^{[1]})^{-1} = -\frac{d\varepsilon_2}{d\varepsilon_1} = -\frac{X_{11}^{[1]}}{X_{22}^{[1]}} \left(\frac{dX_{22}^{[1]}}{d\theta} \right) \left(\frac{dX_{11}^{[1]}}{d\theta} \right)^{-1}, \quad (10)$$

so that after substituting for the derivatives using Eqs. (7) and (8), the equation reduces to:

$$\nu_{12}^{[1]} = (\nu_{21}^{[1]})^{-1} = -1. \quad (11)$$

The Young's moduli can be found from an energy approach. The work W done in changing the angle by $d\theta$ is related to the applied moment by:

$$W = \frac{1}{2} K_h d\theta^2, \quad (12)$$

where since there are eight hinges in each unit cell, the total potential energy U per unit volume, which according to the conservation of energy principle, is equal to the energy due to a strain $d\varepsilon$, is given by:

$$U = \frac{4K_h}{V} d\theta^2 = \frac{1}{2} E_i d\varepsilon_i^2, \quad (13)$$

2.3 The Type β rotating rhombi In the case of Type β networks, the unit cell vectors in Orientation 1 are given by $\mathbf{a}^{[1]} = (X_{11}^{[1]}, 0)$ and $\mathbf{b}^{[1]} = (0, X_{22}^{[1]})$ while those in Orientation 2 are given by $\mathbf{a}^{[2]} = (X_{11}^{[2]}, X_{12}^{[2]})$ and $\mathbf{b}^{[2]} = (0, X_{22}^{[2]})$. The on-axis Poisson's ratios and Young's moduli for this network are most conveniently derived using the unit cell in Orientation 1 (see Fig. 3b) where the projections of the unit cell along the $Ox_1^{[1]}$ and $Ox_2^{[1]}$ directions are respectively given by²:

$$X_{11}^{[1]} = 2a \left[\cos\left(\frac{\theta}{2}\right) + \cos\left(\phi - \frac{\theta}{2}\right) \right], \quad (7)$$

$$X_{22}^{[1]} = 2a \left[\sin\left(\frac{\theta}{2}\right) + \sin\left(\phi - \frac{\theta}{2}\right) \right], \quad (8)$$

where, as illustrated in Fig. 3b, a is the length of the side of the rhombus, ϕ is the internal angle of the rhombi and θ is the angle between the rhombi.

² Note that the angle between the cell vectors for Orientation 2 is given by $\gamma = \phi$ while that in Orientation 1 is given by $\gamma = \pi/2$.

where $V = X_{11}^{[1]} X_{22}^{[1]} X_{33}$ so that on solving for E_i using Eqs. (7)–(9) and assuming a thickness $X_{33} = z$ in the third direction we get:

$$E_1^{[1]} = E_2^{[1]} = \frac{4K_h}{a^2 z \sin^2 \left(\frac{\theta - \phi}{2} \right) \sin \phi}. \quad (14)$$

Furthermore, it can easily be shown that for both orientations the unit cell does not shear, i.e. all the terms in the compliance matrix relating to shear are zero, i.e. the compliance matrix is one which has components $s_{11}^\zeta = s_{22}^\zeta = s_{12}^\zeta = s_{21}^\zeta = 1/E$ and $s_{13}^\zeta = s_{23}^\zeta = s_{31}^\zeta = s_{32}^\zeta = s_{33}^\zeta = 0$, a matrix which remains unchanged when transformed to obtain the off-axis properties, i.e. the mechanical properties of this system are simply given by:

$$\mathbf{S}^{[1]} = \mathbf{S}^{[2]} = \mathbf{S}^\zeta = \frac{1}{E} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where $E = E_1^{[1]} = E_2^{[1]}$.

3 Discussion The derivations presented above show very clearly that varying the way in which the rhombi are connected to each other results in very significant differ-

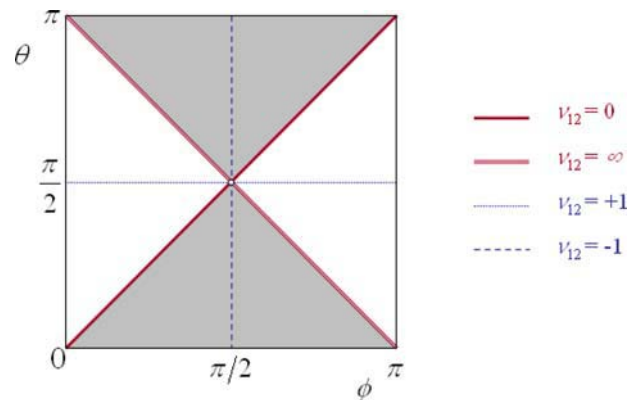


Figure 4 (online colour at: www.pss-b.com) A 2D plot showing all possible combinations of the angles ϕ and θ for the Type α rotating rhombi where the shaded regions indicate those combinations for which on-axis auxeticity is observed in Orientation 2. Note that the hollow circle at $(90^\circ, 90^\circ)$ indicates an undefined point.

ences in the mechanical properties. In fact, whilst the Poisson's ratio of the Type β systems are always negative (i.e. auxetic) with a value of -1 irrespective of:

- the shape and size of the rhombi (i.e. a and ϕ);
- the angle between the rhombi (i.e. θ); and
- the direction of loading (i.e. ζ)

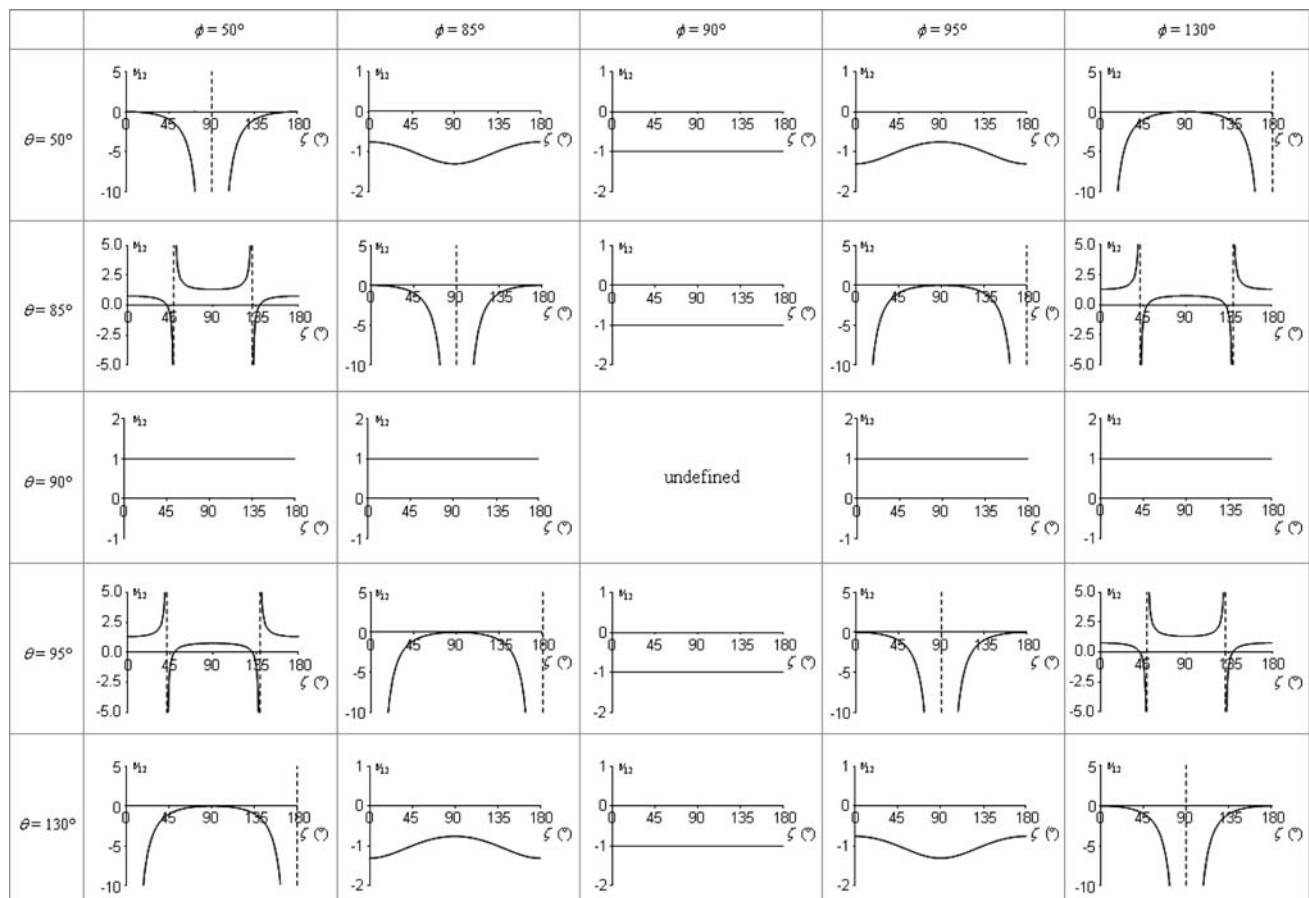


Figure 5 Variation of the Poisson's ratio with the direction of loading for different configurations of the Type α rotating rhombi.

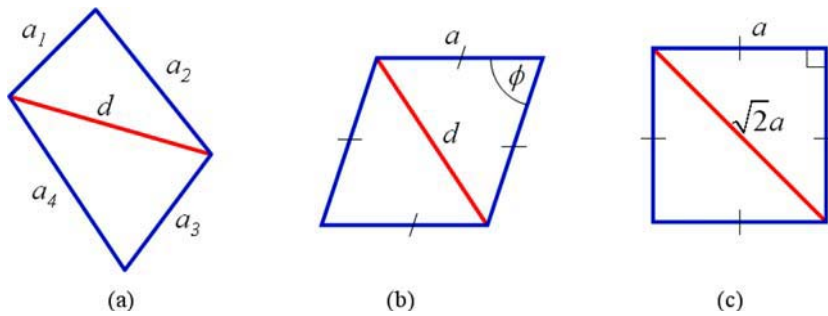


Figure 6 (online colour at: www.pss-b.com) (a) A generalised quadrilateral constructed using four rods, made from the same material, to form the edges and a fifth one having a different thermal expansion coefficient to form a diagonal. (b) A special case of (a) where the sides of the quadrilateral are of equal length and (c) a special case of (b) where the diagonal is so that the internal angles are equal to 90° .

we note that the Type α system can exhibit large variations in the Poisson's ratio which can be either positive or negative, the actual value of which depends on the angles θ , ϕ and ζ (but still independent on the actual size of the rhombi). In fact, the Type β rotating rhombi are characterised by in-plane isotropy and strain independence of the Poisson's ratio. In contrast to this, as illustrated in Fig. 4, the Orientation 2 on-axis Poisson's ratios for the Type α rhombi are negative for half of the combinations of θ and ϕ with the transitions between positive and negative Poisson's ratio occurring when $\theta = \phi$ (where $\nu_{12}^{[2]} = (\nu_{21}^{[2]})^{-1} = 0$) and when $\theta = 180^\circ - \phi$ (where $\nu_{21}^{[2]} = (\nu_{12}^{[2]})^{-1} = 0$). In this respect it is interesting to note that the system with $\phi = 90^\circ$, which corresponds to a system where the rhombi are squares, has Poisson's ratios which always lie in the negative region and are equal to -1 , except at the point when $\phi = \theta = 90^\circ$ for which the Poisson's ratio is undefined (the fully open squares structure). Also, the system when $\theta = 90^\circ$ (i.e. the system when the empty space between the rhombi is a square which may be considered as the 'negative' of these rotating squares system) has Poisson's ratios which are always positive and equal to $+1$ except when $\phi = \theta = 90^\circ$ where once again the Poisson's ratio is undefined.

From a practical point of view, it is important to note that all structures based on the 'rotating quadrilaterals' principle can be assembled from quadrilaterals constructed using five pin-jointed truss elements four of which form the sides of the quadrilateral, with the fifth forming a diagonal as shown in Fig. 6. The case of 'rotating rhombi',

therefore, corresponds to systems where the four elements making the side of the rhombi are of equal length a and the fifth diagonal element has length d where $0 < d < 2a$ whilst in the more special case of the 'rotating squares', the diagonal element must have a length which is equal to $\sqrt{2}a$. As illustrated in Fig. 7, there are two ways how such systems may be connected: either with the diagonal elements touching each other, which will result in a Type β system, or with the diagonal elements never touching each other which will result in a Type α system. The mechanical properties for such systems may be re-written in terms of the lengths a and d and the angle θ where if we assume that length d is opposite to the angle ϕ we find that the angle ϕ can be related to a and d by:

$$\sin\left(\frac{\phi}{2}\right) = \frac{d}{2a} \quad \text{and} \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{4a^2 - d^2}}{2a} \quad (15)$$

or alternatively as

$$\sin(\phi) = \frac{d\sqrt{4a^2 - d^2}}{2a^2} \quad \text{and} \quad \cos(\phi) = 1 - \frac{d^2}{2a^2} \quad (16)$$

so that the unit cell projections for the Type α system can be rewritten as:

$$X_{11}^{[2]} = \sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right) + d \cos\left(\frac{\theta}{2}\right), \quad (17)$$

$$X_{22}^{[2]} = \sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right) + d \sin\left(\frac{\theta}{2}\right), \quad (18)$$

from which it follows that the mechanical properties can be given by:

$$\nu_{12}^{[2]} = \frac{\left(d \cos\left(\frac{\theta}{2}\right) - \sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right)\right) \left(d \cos\left(\frac{\theta}{2}\right) + \sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right)\right)}{\left(d \sin\left(\frac{\theta}{2}\right) + \sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right)\right) \left(d \sin\left(\frac{\theta}{2}\right) - \sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right)\right)}, \quad (19)$$

$$E_1^{[2]} = \frac{16K_h \left(\sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right) + d \cos\left(\frac{\theta}{2}\right)\right)}{z \left(\sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right) + d \sin\left(\frac{\theta}{2}\right)\right) \left(\sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right) - d \sin\left(\frac{\theta}{2}\right)\right)^2}, \quad (20)$$

$$E_2^{[2]} = \frac{16K_h \left(\sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right) + d \sin\left(\frac{\theta}{2}\right) \right)}{z \left(\sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right) + d \cos\left(\frac{\theta}{2}\right) \right) \left(d \cos\left(\frac{\theta}{2}\right) - \sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right) \right)^2}. \quad (21)$$

Similarly, for Type β systems

$$X_{11}^{[1]} = \frac{4a^2 - d^2}{a} \cos\left(\frac{\theta}{2}\right) + \frac{d\sqrt{4a^2 - d^2}}{a} \sin\left(\frac{\theta}{2}\right), \quad (22)$$

$$X_{22}^{[1]} = \frac{d^2}{a} \sin\left(\frac{\theta}{2}\right) + \frac{d\sqrt{4a^2 - d^2}}{a} \cos\left(\frac{\theta}{2}\right), \quad (23)$$

so whilst the Poisson's ratio is still given by -1 , the Young's moduli for such systems are given by:

$$E = \frac{32a^2 K_h \left[d^2 \sin\left(\frac{\theta}{2}\right) + d\sqrt{4a^2 - d^2} \cos\left(\frac{\theta}{2}\right) \right]}{z \left((4a^2 - d^2) \cos\left(\frac{\theta}{2}\right) + d\sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right) \right) \left(d^2 \cos\left(\frac{\theta}{2}\right) - d\sqrt{4a^2 - d^2} \sin\left(\frac{\theta}{2}\right) \right)^2}. \quad (24)$$

These constructions, where the rhombi are built using rods rather than plates, introduces the possibility to have systems which can respond to temperature changes and thus, in some cases, have temperature dependent Poisson's ratios. In fact, if the diagonal rod is made up from a material which has a different thermal expansion coefficient from the rods making up the sides, the length of the diagonal changes by a different amount than the sides with the result that the rhombi change shape, and thus their Poisson's ratio, if the diagonals are connected in a form corresponding to a Type α structure. Note that similar effects have been used elsewhere to produce systems exhibiting negative thermal expansion [34–37], a property which incidentally may also be exhibited by our system.

To quantify these effects, we will extend the model by making the lengths a and d of the system to be temperature dependent. In particular, assuming that the rods are uniform and the temperature changes are such that the change in length of the rods can be assumed to be directly proportional to the linear thermal expansion coefficient α over the temperature range considered, then if the temperature of the system is changed from a reference temperature T_0

where the lengths of the sides and diagonals are a_0 and d_0 respectively to a temperature T where the lengths of the sides and diagonals are a and d respectively, then the lengths a and d are related to T , T_0 , a_0 and d_0 through:

$$a = a_0 [1 + \alpha_a (T - T_0)], \quad (25)$$

$$d = d_0 [1 + \alpha_d (T - T_0)], \quad (26)$$

where α_a and α_d are linear thermal expansion coefficients of the elements making the sides and the diagonals respectively.

These equations, when substituted in Eqs. (18)–(21) may be used to predict the extent to which a temperature change affects the mechanical properties and suggest that whilst the Poisson's ratio for the Type β system is unaffected by temperature (it is always equal to -1), the Poisson's ratio for the Type α is affected by the initial configuration of the system, the thermal expansion coefficients and the temperature. To illustrate these variations for the Type α system in a realistic manner we use thermal expansion coefficient values typical of polymers, in particular

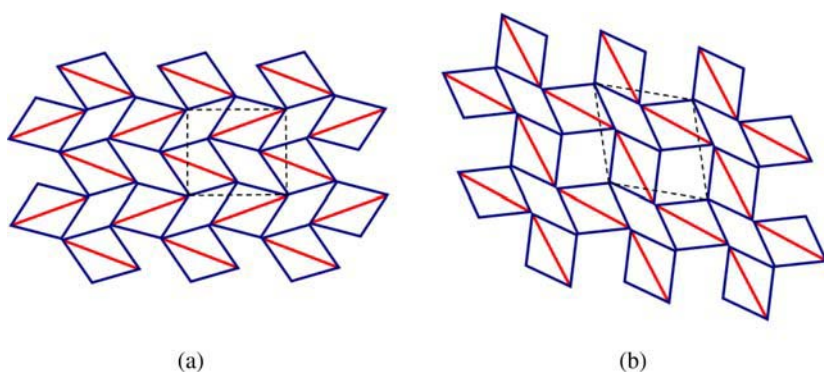


Figure 7 (online colour at: www.pss-b.com) Units like the one shown in Fig. 5b can be connected with the diagonals of adjacent units (a) not connected to each other to give the Type α rotating rhombi or (b) connected to each other to give the Type β rotating rhombi.

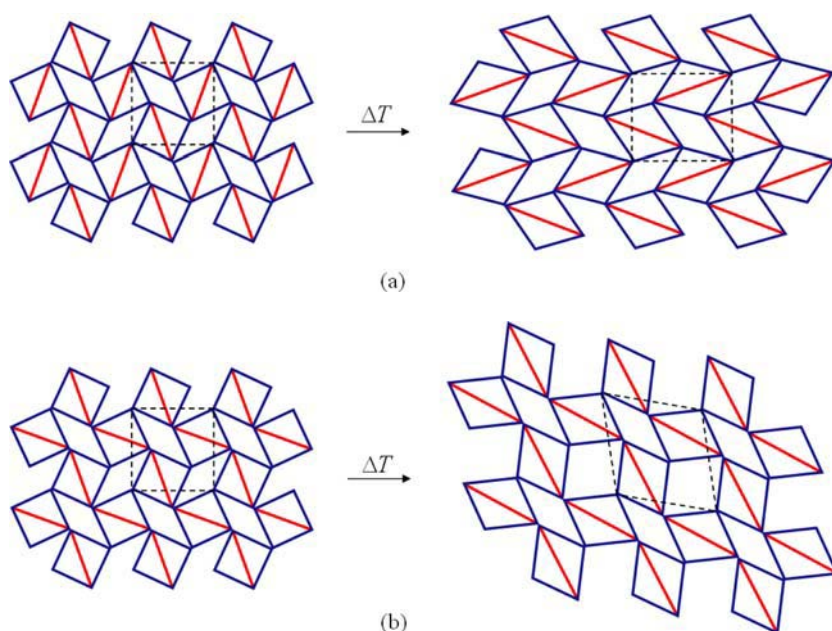


Figure 8 (online colour at: www.pss-b.com) Systems constructed using the unit in Fig. 5c deform to (a) Type α rotating rhombi if the diagonals of adjacent squares are not connected to each other or to (b) Type β rotating rhombi if the diagonals of adjacent squares are connected to each other.

$80 \times 10^{-6} \text{ K}^{-1}$ and $180 \times 10^{-6} \text{ K}^{-1}$ for α_a and α_d respectively, to plot the variation of the on-axis Poisson's ratios with the angle θ for different initial configurations at temperatures $T = T_0$ and $T = T_0 \pm 100 \text{ K}$. These plots (Figs. 9 and 10) clearly suggest that although the effect of a temperature change on the Poisson's ratio is in some cases minimal, there is a significant change in the magnitude of the Poisson's ratio for geometries which are such that the diagonal tends to its maximum permitted value, particularly in close proximity to the region where the Poisson's ratio changes

sign asymptotically i.e. when $\theta = 180^\circ - \phi$ in the case of $\nu_{12}^{[2]}$ and when $\theta = \phi$ in the case of $\nu_{21}^{[2]}$. In this respect, it is interesting to note that the cases in close proximity to $\theta = \phi$ and $\theta = 180^\circ - \phi$ are always of particular interest since any small changes in the geometry may result in a change of the sign of the Poisson's ratios $\nu_{12}^{[2]}$ and $\nu_{21}^{[2]}$. All this is very significant as we have, for the first time, showed a simple way how systems can be made to exhibit temperature dependent Poisson's ratios, which can be positive or negative.

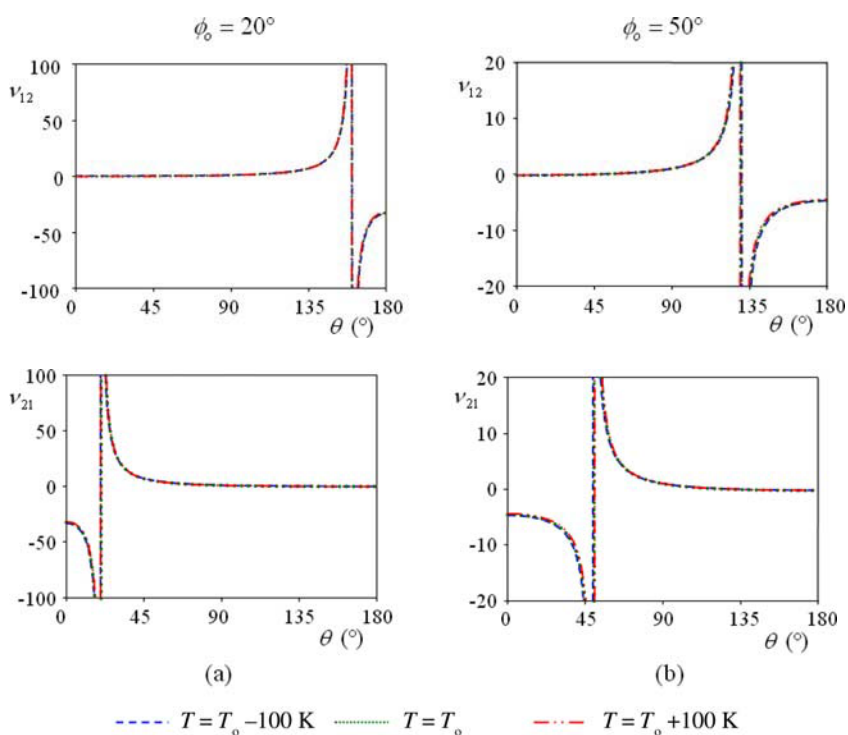


Figure 9 (online colour at: www.pss-b.com) Plots showing the variation of the Poisson's ratio as a function of θ (the degree of openness of the system) at different temperatures for rhombi having an internal angle of (a) 20° and (b) 50° .

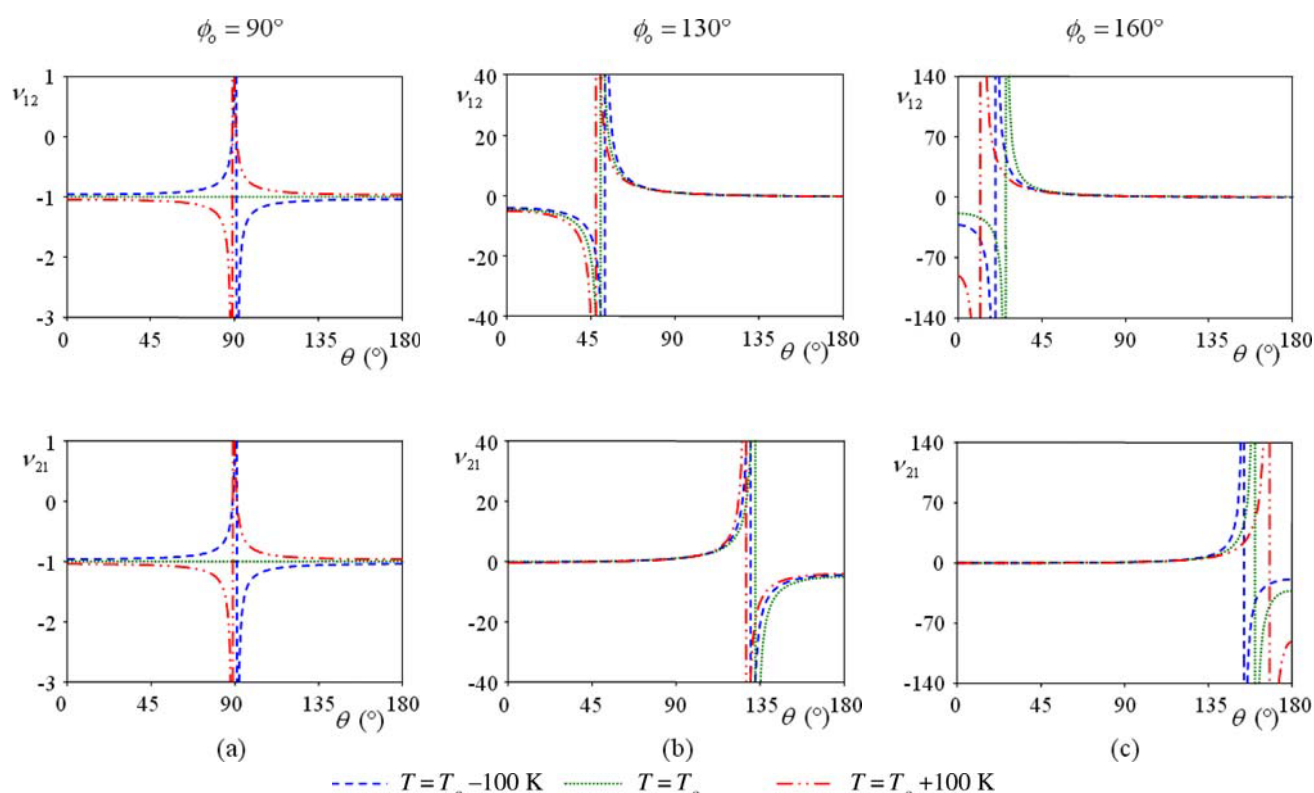


Figure 10 (online colour at: www.pss-b.com) Plots showing the variation of the Poisson's ratio as a function of θ (the degree of openness of the system) at different temperatures for rhombi having an internal angle of (a) 90° (i.e. for rotating squares), (b) 130° and (c) 160° .

4 Conclusion In this work, we reviewed how rhombi can be connected in two distinct ways to obtain either Type α or Type β rotating rhombi and we explored how these may be constructed using pin-jointed rigid rods. We showed that these two different configurations exhibit very distinct mechanical properties. In particular, as discussed elsewhere, the Type α rotating rhombi can exhibit both auxetic and conventional behaviour depending on the initial configuration of the systems (i.e. θ and ϕ) and also on the direction of loading (ζ), whereas Type β rotating rhombi discussed here are always auxetic with an isotropic Poisson's ratio of -1 . In addition to this we have also shown that the mechanical properties of the Type α systems can be made temperature dependent by constructing the rhombi from pin-jointed rods where the material used to form the sides of each rhombus has a different thermal expansion coefficient to the rods used to make the diagonals. This behaviour compliments earlier work [38–44] on molecular models consisting of cyclic hexamers, heptamers and trimers made from ‘hard discs’ where the Poisson's ratio for such models changes smoothly [38–42] and in some cases also discontinuously [43, 44] with variation in thermodynamic parameters such as density, pressure/stress (at constant temperature) or temperature (at constant pressure).

In particular, in this work we have shown that the temperature dependence of the Poisson's ratio of our mechani-

cal systems becomes very pronounced near the region where the diagonal approaches its maximum permitted length of twice the length of the side of the rhombus where Poisson's ratio changes sign discontinuously. This is in contrast to Type β systems whose Poisson's ratios are temperature independent.

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