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Global rotation of mechanical metamaterials induced by their internal deformation

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In this work, we propose the concept that a device based on mechanical metamaterials can be used to induce and control its own rotational motion as a result of internal deformations due to the conversion of translational degrees of freedom into rotational ones. The application of a linear force on the structural units of the system may be fine-tuned in order to obtain a desired type of rotation. In particular, we show, how it is possible to maximise the extent of rotation of the system through the alteration of the geometry of the system. We also show how a device based on this concept can be connected to an external body in order to rotate it which result may potentially prove to be very important in the case of applications such as telescopes employed in space. https://doi.org/10.1063/1.4998398

I. INTRODUCTION

Mechanical metamaterials are systems whose anomalous mechanical properties are derived primarily from their geometry rather than material composition.[1,2] This revolutionary class of functional materials has attracted a great deal of interest in recent years due to the fact that mechanical metamaterials may be tailor-made to exhibit counter-intuitive properties such as a negative Poisson's ratio,[3–7] negative compressibility,[8,9] negative stiffness[10,11] and negative thermal expansion,[12,13] all of which are rarely found in natural or conventional everyday materials. These wondrous properties make these systems ideal for a variety of applications ranging from biomedical[14,15] to acoustic[16,17] and protective devices.[3,18]

Despite the numerous studies conducted on these systems in recent years, many of their aspects remain unexplored. One of these aspects is the potential of the whole system to exhibit a rotational motion upon the application of a uniaxial stress/strain. This effect, apart from being interesting from the theoretical point of view, may also be used for the attitude control of the external body.

In recent years, the attitude control of free body systems has been often attained by the use of reaction wheels.[19–21] These structures are incorporated with the free body and rotate as a result of a direct application of a torque by means of a motor. This in turn, due to the conservation of angular momentum principle, leads to the overall rotation of the system. However, the direct application of a torque is not the sole method which may lead to the rotation of the system. As we will show here, such effect may also be obtained in certain specific systems through the application of a uniaxial force or strain which induces rotation of the individual components constituting the reaction mechanism. Mechanical metamaterials are one such class of systems where internal rotational motion may be induced through the application of a uniaxial force. Indeed

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there is a whole field of study on a group of auxetic[6] (negative Poisson's ratio) mechanical metamaterials which are called rotating unit systems,[22,23] which as their name suggests, deform via rotations of the respective components of the system. These internal rotations could in theory also be used to produce a reaction torque resulting with a change in the orientation of the structure in space.

In view of this, in this work, we present a proof of concept of how mechanical metamaterials may be used to induce a rotation of the structure as result of internal deformations resulting from the application of the uniaxial force. We also show how the extent of this effect can be controlled by a variety of geometric parameters corresponding to the system.

II. MODEL

In this section, a model designed to induce and control its own rotational motion based on the two-dimensional rotating rigid squares mechanism will be presented. This auxetic mechanism, which is one of the earliest systems studied with respect to its potential to exhibit a negative Poisson's ratio, consists of square units connected to each other at their corners through hinges (see Fig. 1(a)). When the rotating squares structure is uniaxially stretched, the individual squares constituting the structure rotate relative to each other in order to attain a more open conformation, with every square rotating in the opposite direction to the one adjacent to it.

At this point, it is important to note that rigid units which in this work are referred to as squares, in reality represent cuboids connected in exactly the same manner as squares shown in Fig. 1(a). This nomenclature is used in order not to confuse the discussed structure with the other well-known mechanical metamaterial which is often referred to as rotating cuboids system[24] which system corresponds to a completely different deformation mechanism.

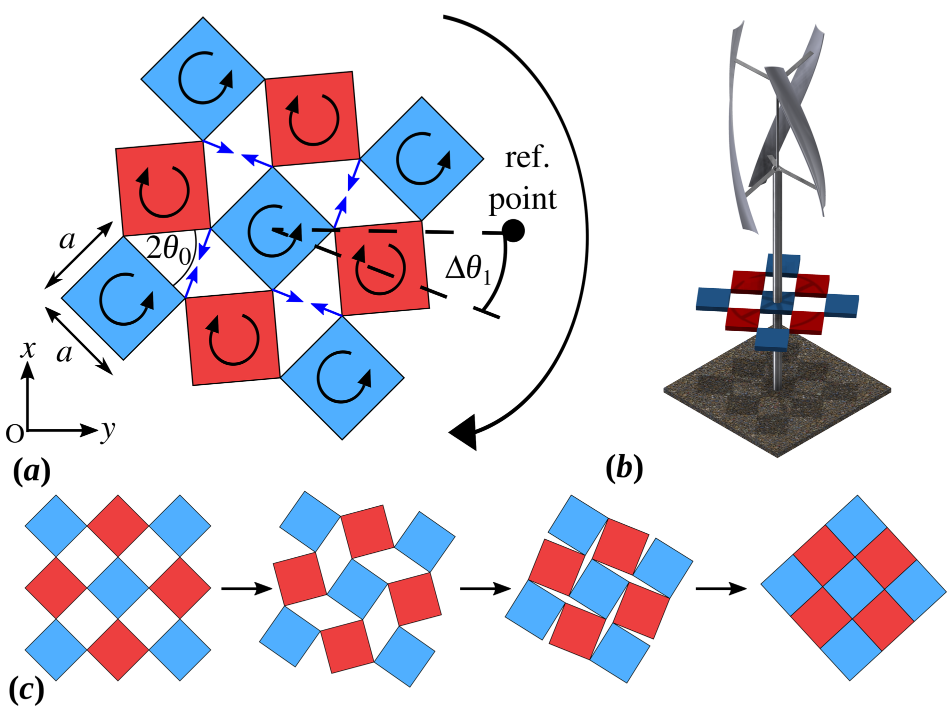

FIG. 1. The panels show (a) the model of the discussed system with schematically drawn blue arrows indicating the positioning of linear actuators inducing a deformation of the system (black arrows indicate all types of rotations exhibited by the system), (b) a diagram presenting a possible connection of the discussed system with an external body, (c) diagrams presenting the behaviour of the system in which the rotation of rigid units results with a decrease of the angle θ_0 and a change in the value of $\Delta\theta_1$. The change in the value of $\Delta\theta_1$ corresponds to the rotation of the structure with respect to the external reference system which is schematically denoted as O.

In this work, a three-dimensional system which cross-section may be described as a finite rotating rigid square system made up of $N_S \times N_S$ squares will be considered where the rigid units are connected at vertices acting as hinges. Deformation of the system is assumed to occur only through opening and closing of these hinges, with squares themselves remaining perfectly rigid. The cross-sectional dimensions of the rotating units, i.e. the lengths of their sides, are defined by the parameter a, while their thickness or depth is denoted by z. Consequently, their mass, M, may be defined in terms of these dimensions and the density of the rotating units, ρ, as: $M = \rho z a^2$.

In this work, it is assumed that the rigid units constituting the system rotate with the constant angular acceleration ε_0 which results with a change in the angle between the adjacent units. This angle is denoted as $2\theta_0$ (see Fig. 1(a)) which means that the respective units rotate with the angular velocity $\omega_0 = \frac{d\theta_0}{dt}$. It is also worth to note that the rigid units may rotate within the limits of geometric constraints of the system, i.e. as long as the condition $\theta_0 \in [0°, 90°]$ is satisfied. In order to induce the rotation of respective rigid units in a way described above, one could use the set of linear actuators embedded in a system in a way so that the opposite ends of diagonals of apertures formed between the adjacent units would be either brought closer or farther from each other (see Fig. 1(a)).

In this work, the respective rigid units within the system rotate with an angular velocity changing in accordance to a constant angular acceleration. This process leads to the change in their angular momentum in time which results with a generation of a torque by each of the units. Since in a rotating square system each square rotates in the direction opposite to the one next to it, the net torque of a system made up of an even number of identical rotating units should be equal to zero, as the opposing rotations of the adjacent individual units comprising the system would cancel each other out. However, if N_S is an odd number, the number of squares rotating in one direction will be greater by one in comparison to those rotating in the opposite direction. In view of this, systems considered in this work will only correspond to odd values of N_S as such systems have the potential to generate a larger overall torque than their even-numbered counterparts which would result with a greater extent of rotation. Another way of amplifying the net torque generated by the system corresponds to the differentiation of the mass of two sets of units rotating in the opposite directions, with squares in different sets being equally-sized but having a different density. The masses of these squares will be denoted by M_H and M_L respectively, where M_H and M_L correspond to units with larger (ρ_H) and lower density (ρ_L) respectively. The square at the centre of the system will always be assumed to have a mass of M_H and therefore the system will in all cases be made up of $\frac{N_S^2+1}{2}$ heavy rotating units and $\frac{N_S^2-1}{2}$ light rotating units. Furthermore, the extent of rotation of the whole system with respect to its centre of mass and an external global axis will be denoted by $\Delta\theta_1$, where initially the unrotated system has a θ_1 value of 0°.

As it was mentioned above, the accelerated motion of respective heavy and light units results with a generation of the net torque τ_0 $\left(\tau_0 = \left|\vec{\tau_0}\right|\right)$, which quantity may be defined in the following manner:

$$\tau_0 = \frac{dL_H}{dt} - \frac{dL_L}{dt} + \frac{dL_{ob}}{dt}, \tag{1}$$

where, L_H and L_L stand for a sum of angular momenta coming from all of the heavy and light units respectively. L_{ob} represents the magnitude of the angular momentum associated with an external body attached to the centre of the square located in the middle of the system. As shown in Fig. 1, the centre of the square in the middle of the system also corresponds to the axis of rotation of the whole system. The third term in equation 1 has the same sign as the term corresponding to the rotation of heavy units, which stems from the fact that the external body is attached to the heavy square. As a result, it must also rotate with the same angular velocity as heavy units which rotate with respect to their own centres (see Fig. 1(a)). At this point, one should note that in the case when there is no external body attached to the system constituted by rigid squares, the last term in the above equation assumes the value of 0. Eq.(1) may also be written down in a discrete form in terms of parameters corresponding to the mass distribution and geometry of the system (see supplementary material).

Torque τ_0 contributes to the overall rotation of the system with respect to its centre of mass, which in turn is associated with the change in the angle θ_1. Taking all of this into consideration, an overall rotation of the discussed system, induced by the opening/closing of the rotating units, can be expressed through the rotational analog of Newton's equation of motion in the following manner:

$$-\tau_0 + \tau_{ext} = \frac{d}{dt}\left[(I_1 + I_{ob})\frac{d\theta_1}{dt}\right], \tag{2}$$

where the negative sign in front of τ_0 arises due to Newton's third law for rotational motion as the magnitude of reaction torque has the same magnitude as torque τ_0 generated by individual units but the opposite orientation. Torque τ_{ext} is associated with any additional factors which may affect the overall rotational motion of the system, i.e. factors such as an additional motor located on the main axis of rotation which directly induces a rotation of the system, wind, air resistance etc. Assuming that there are not any additional factors contributing to the global rotation of the system, the term τ_{ext} would assume the value of 0. The moment of inertia I_1, corresponds to the rotation of all of the rigid units with respect to the centre of mass of the whole system and may be defined as follows (see supplementary material for the full derivation):

$$I_1 = \frac{1}{12}a^2\left[\left(N_S^2 - 1\right)M_L + \left(N_S^2 + 1\right)M_H\right] + \frac{1}{4}d^2\left(N_S^2 - 1\right)\left[\left(\frac{N_S^2}{3} - 1\right)M_L + \left(\frac{N_S^2}{3} + 1\right)M_H\right], \tag{3}$$

where, d stands for the distance between centres of adjacent squares and can be expressed by means of the following equation: $d = \sqrt{2}a\sin\left(\frac{\pi}{4} + \theta_0\right)$. One should also note that based on the formulas defining I_1 and d it is possible to express Eq. (2) in a more explicit manner as shown in supplementary material.

III. RESULTS AND DISCUSSION

In order to analyse the behaviour of the discussed system, Eq.(2) was solved numerically by means of the fourth-order Runge Kutta algorithm.[32] In this work, all of the results were generated under the assumption that the auxetic system defined in the model section is being deformed from its fully-open to the fully-closed conformation, which corresponds to the change in the value of $2\theta_0$ from 90° to 0°. The parameters used in order to generate these results, which were the same for all considered sets of results, were set to be the following: $\tau_{ext} = 0$ Nm, $a = 0.33$ m, $z = 0.02$ m, $\rho_H = 8000$ kg m^{-3}, $M_H = 17.78$ kg, $\rho_L = 2000$ kg m^{-3}, $M_L = 4.44$ kg, $I_{ob} = 0$ kg m^2, $\varepsilon_0 = -0.5$ rad s^{-2}, $\omega_0(t = 0\,\text{s}) = 0$ deg s^{-1}.

Results shown in Fig. 2(a) were generated in order to determine which value of N_S results with the maximum enhancement of the extent of rotation of the investigated system with respect to its centre of mass. In order to do that, the value of N_S was set to assume the respective values from the given set of numbers $\{3, 5, 7, 9\}$ with the remaining parameters being set to be the following: $a \in \{0.33, 0.2, 0.143, 0.11\}$m, ρ_H = 8000 kg m^{-3}, $M_H = \{17.78, 6.4, 3.265, 1.975\}$ kg, ρ_L = 2000 kg m^{-3}, $M_L = \{4.44, 1.6, 0.816, 0.494\}$ kg. Furthermore, results shown in Fig. 2(b) were generated in order to investigate a change in the behaviour of the system upon varying the magnitude of ρ_H/ρ_L ratio. In the case of all of the considered values of these ratios, the total mass of the system and dimensions of rigid units were kept constant. The remaining parameters used in order to generate these results were set as follows: $\rho_H/\rho_L \in \{1, 2, 3, 4, 5, \infty\}$, $\rho_H \in \{5333.33, 6857.14, 7578.95, 8000.0, 8275.86, 9600.0\}$ kg m^{-3}, $M_H \in \{11.852, 15.24, 16.84, 17.77, 18.39, 21.33\}$ kg, $\rho_L \in \{5333.33, 3428.57, 2526.32, 2000.0, 1655.17, 0.0\}$ kg m^{-3}, $M_L \in \{11.85, 7.62, 5.61, 4.44, 3.67, 0.0\}$ kg.

Based on Fig. 2, one may note that the rotation of respective rigid units may result with the global rotation of the whole system which behaviour is clearly manifested by nonzero values of $\Delta\theta_1$ and ω_1 on the graphs. This means that in order to induce the rotation of the system, there is no need for the external application of the force to the system which process would result with the generation of the torque. This stems from the fact that the torque contributing to the rotation of the system with respect

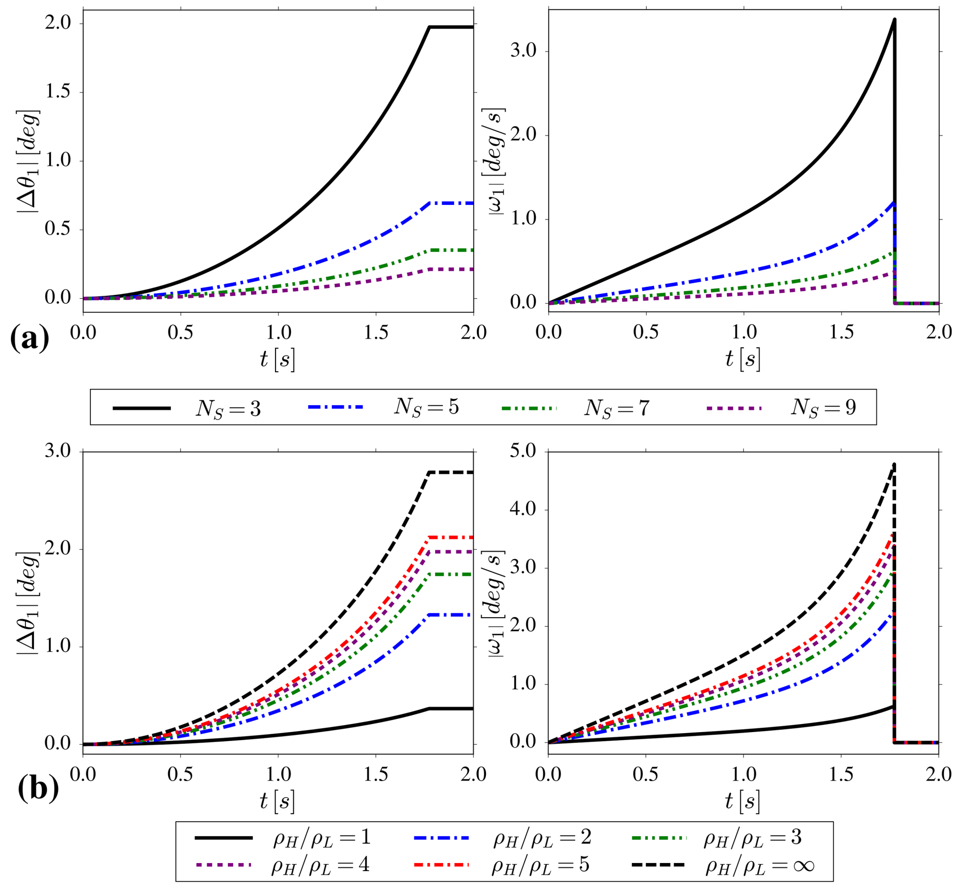

FIG. 2. The panels show (a) comparison of the behaviour of systems consisting of a different number of rigid units and (b) the change in the behaviour of the system upon varying the magnitude of the ratio of densities of heavy and light units for a system with a conserved mass. The point where the system stops exhibiting the global rotation, i.e. the values of $\Delta\theta_1$ stop changing, corresponds to the conformation of the system where $2\theta_0 = 0°$.

to its centre of mass may be generated by the change in the angular velocity of its constituents as explained in the model section.

Apart from the fact that the rotation of respective rigid units constituting the discussed system may result with its overall rotation, it is also interesting to consider an optimisation of this effect. More specifically, it is interesting to check which parameters associated with the geometry and mass distribution within the system could enhance the extent of rotation of the investigated system.

As discussed previously in the model section, variation of the mass of the two sets of rotating units rotating in opposite directions also affects the torque experienced by the whole system when subjected to a linear deformation resulting in the rotation of the respective units. In Fig. 2(b), results are shown for systems having the same total mass as well as a size and angular velocity of rigid units (associated with a constant value of ε_0), with different density ratios, ρ_H/ρ_L. It is evident from the plots that the larger the difference between the masses of the two sets of rotating units, the greater the extent of rotation of the system. However, while there is a large difference between plots of the angular velocity generated for systems where the ρ_H/ρ_L ratio is 1 and 2, the difference between systems corresponding to consecutive values of ρ_H/ρ_L decreases significantly, indicating that the effect of this parameter on the extent of rotation of the system becomes comparable for relatively large values. Another parameter that has a significant effect on the magnitude of the discussed effect, is the value of N_S. As shown in the plots in Fig. 2(a), the maximum extent of rotation and the corresponding angular velocity were generated by the system with the smallest number of rotating units, i.e. N_S = 3. This is very convenient since it means that there is no need to design a structure with a large number of rotating units and hence, a large number of small actuators, which could make the system more prone to malfunctions and defects, in order to generate a large reaction torque.

At this point it is also important to highlight the fact that the discussed system may also affect the rate of rotation of the external system without being deformed, i.e. when the respective rigid units constituting the system are not rotating. This result, which is not normally observed in the case of other devices allowing to induce the rotation of the external system such as reaction wheels, stems from the fact that even when the respective units stop rotating the new conformation of rotating squares varies from the initial one. This means that the whole system corresponds to a different moment of inertia. This in turn can make it either simpler or more difficult to rotate the system depending on the value of the moment of inertia.

All this is very significant, since these results show that the novel metamaterial-based device presented here, besides being an effective alternative method for attaining the control over the rotation of the system, is also extremely versatile since it allows to fine-tune the extent of rotation of the system by varying a number of parameters. Moreover, the rotation of the entire system is induced through the application of tensile force on the sub-structure of the mechanical metamaterial rather than through a direct application of a torque to the rigid body. At this point it must be emphasized that the mathematical model presented in this work is merely one example of this new class of rotational motion controllers and in general one could use other mechanical metamaterials deforming via the rotation of its subunits.[25–31]

Before concluding, it is important to highlight the potential applicability of these systems. As mentioned previously, the control over the rotational motion is one of the most important factors in attitude control of spacecraft. Moreover, these mechanical metamaterial-based systems could also be employed in concert with other systems to fine-tune the orientation of objects such as telescopes. Another potential use for these systems is in wind turbines (see Fig. 1(b)). The efficacy of wind turbines for the production of energy depends strongly on the angular velocity of the system, with maximum efficiency being achieved if the optimal angular velocity is maintained at all times. However, in reality, shifting wind currents make this extremely difficult, and thus the discussed device could be implemented within the turbine in a manner such as that shown in Fig. 1(b) in order to increase/decrease the moment of inertia depending on a strength of wind. This in turn would make it significantly simpler for rotating blades to maintain a particular value of the angular velocity of the wind turbine.

IV. CONCLUSION

In conclusion, in this paper, a proof of concept of how uniaxial loading of mechanical metamaterials may be used to induce a global rotation of the whole system was presented. A novel model designed to predict and quantify this phenomenon was also presented. It was also shown that upon varying the magnitude of parameters associated with the system it is possible to induce the rotation of the system to varying extents. Another interesting result reported in this work corresponds to the possibility of affecting the rate of rotation of the external body by a change in the moment of inertia of the discussed system. This is very significant as it suggests that this model can be used to control the rotation of a wide range of currently employed mechanisms in different branches of industry and science. Some of the most prominent examples of such applications are satellites, cosmic telescopes and wind turbines where different solutions concerning the control of the rotational motion are already employed. In view of this it is hoped that this work will lead to further interest from scientists in the dynamics and physical properties of mechanical metamaterials as well as eventually lead to the industrial production and implementation of devices based on this concept.

SUPPLEMENTARY MATERIAL

See supplementary material for a detailed derivation and description of certain concepts presented in this work.

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